A Branch-and-Cut Algorithm for Energy-Aware Job-Shop Scheduling

under Time-of-Use Pricing and a Peak Power Limit

Marouane Felloussi, Xavier Delorme, Paolo Gianessi

Mines Saint-Etienne, LIMOS UMR 6158 CNRS,
Saint-Etienne, France

February 28, 2025

ROADEF 2025, Champs-Sur-Marne, France

Outline

Outline

Energy-Aware Job-Shop Scheduling

The Job-Shop Scheduling Problem:

- Schedule jobs with ordered operations on machines to optimize some criterion.
- Relevant problem in OR, strongly NP-hard [1].
- Modeling Approaches:
  - » Time-Indexed formulations $\Rightarrow$ strong dual bounds, large MILPs.
  - » Disjunctive formulations $\Rightarrow$ more compact MILPs, weak relaxations.

Energy-Aware Job-Shop Scheduling

Energy consideration:

- Production is energy-intensive, electrification of processes (etc.)
- ToU pricing:
  - » Non-regular criterion.
  - » Period-Indexed formulation.
- Peak power limit:
  - » Energy: cumulative resource.
  - » Machines: unitary resources.

Figure: Seasonal Time-of-Use pricing example [2]

Problem Definition

- **Machines** $m \in \mathcal{M}$ with power consumption φ_m.
- **Jobs** $j \in \mathcal{J}$ to execute over a time horizon C:
  - » Processed over an ordered subset of machines.
  - » Having constant processing times $q_{j,m} \Rightarrow$ energy consumption $\varphi_m \cdot q_{j,m}$.
  - » Direct precedence constraints.
- **Operations** $o \in \mathcal{O} := \mathcal{J} \times \mathcal{M}$, i.e. processing of a job on a machine.
- **ToU Periods** $p \in \mathcal{P}$:
  - » Duration l^p and electricity price c_p per unit.
  - » $[t^p, t^{p+1}]$, with $t^{p+1} - t^p = l_p$, $t^1 = 0$ and $t^{|\mathcal{P}|} = C$.
- **Peak Power Limit** $\overline{\varphi}$ to respect at all times.

Problem Definition

A **feasible solution** consists of a **schedule** where each job $j \in \mathcal{J}$ processes over machines $m \in \mathcal{M}$, during one or more ToU periods in $\mathcal{P}$, such that:

- same-machine operations do not process simultaneously **(non-overlap)**,
- operations may not interrupt processing **(non-preemption)**,
- operation sequencing respects a predefined order **(precedence)**.
- the total power usage of all machines never exceeds the limit **(capacity)**.

Objective

The goal is to find a **feasible** solution **minimizing** the Total Energy Cost **(TEC)**.

Instance and Solution

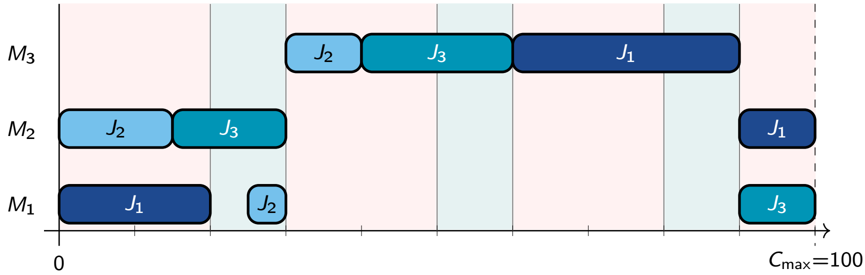
Machine m	1	2	3
Power φ_m	5	6	8

Job j	1	2	3
Sequence M_j	$\{1,3,2\}$	$\{2,1,3\}$	$\{2,3,1\}$

Machine m	1	2	3
$q_{1,m}$	20	10	30
$q_{2,m}$	5	15	10
$q_{3,m}$	10	15	20

ToU period p	on-peak	off-peak
Tariff c^p	0.159	0.13
Duration l^p	20	10

Makespan minimization s.t. $\overline{\varphi} = 13$ (TEC $= 204.7$)



Instance and Solution

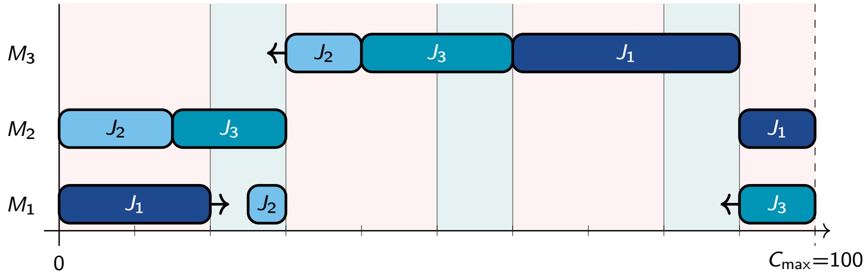
Machine m	1	2	3
Power φ_m	5	6	8

Job j	1	2	3
Sequence M_j	$\{1,3,2\}$	$\{2,1,3\}$	$\{2,3,1\}$

Machine m	1	2	3
$q_{1,m}$	20	10	30
$q_{2,m}$	5	15	10
$q_{3,m}$	10	15	20

ToU period p	on-peak	off-peak
Tariff c^p	0.159	0.13
Duration l^p	20	10

Makespan minimization s.t. $\overline{\varphi} = 13$ (TEC = 204.7)



Instance and Solution

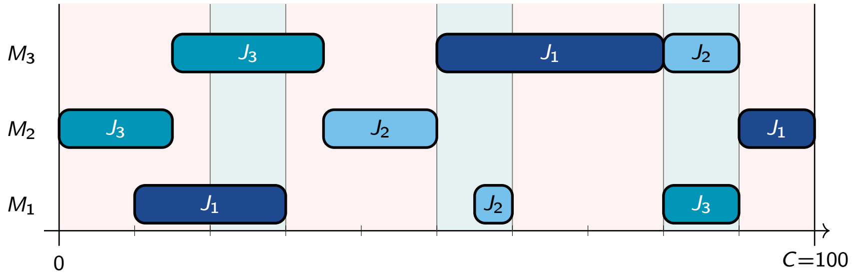
Machine m	1	2	3
Power φ_m	5	6	8

Job j	1	2	3
Sequence M_j	$\{1,3,2\}$	$\{2,1,3\}$	$\{2,3,1\}$

Machine m	1	2	3
$q_{1,m}$	20	10	30
$q_{2,m}$	5	15	10
$q_{3,m}$	10	15	20

ToU period p	on-peak	off-peak
Tariff c^p	0.159	0.13
Duration l^p	20	10

Energy Cost minimization s.t. $\overline{\varphi} = 13$ and $C_{\max}$ (TEC $= 197.1$)



Short literature review

Problem class	Article	Problem*	Solution Approach
job-shop scheduling	[4]	$Jm\|\|$TEC	MILP (PI)
	[5]	$Jm\|$ on/off, r_j, $d_j\|$TEC	MILP (TI), B&B
	[6]	$Jm\|\mathrm{P}_{\max}\|$TEC	MILP (D,TI), MH
flexible job-shop scheduling	[7]	$FJm\|$ on/off $\|C_{\max}$,TEC	MILP (TI), CP
	[8]	$FJm\|\|C_{\max}$,TEC	MILP (TI), H
flow-shop scheduling	[9]	$F2\|prmu$, on/off $\|$TEC	MILP (PI+TI), LBBD
parallel machine scheduling	[10]	$Pm\|\|C_{\max}$,TEC	MILP (TI), H
	[11]	$Rm\|\|$TEC	MILP (PI), H
	[12]	$Rm\|\|$TEC	MILP (PI), B&P
single machine scheduling	[13]	$1\|batch\|$TEC	MILP (PI,TI), CG-H
	[14]	$1\|batch\|$TEC	MILP (PI)
RCPSP	[15]	$PS\|prec\|C_{\max}$,TEC	MH
	[16]	$PS\|prec\|$TE	MILP (EB)

Table: An overview of related works.

*Graham's 3-field notation [3]

Short literature review

Problem class	Article	Problem*	Solution Approach
job-shop scheduling	[4]	$Jm\|\|\text{TEC}$	MILP (PI)
	[5]	$Jm\|\text{on/off}, r_j, d_j\|\text{TEC}$	MILP (TI), B&B
	[6]	$Jm\|\text{P}_{\text{max}}\|\text{TEC}$	MILP (D,TI), MH
flexible job-shop scheduling	[7]	$FJm\|\text{on/off}\|C_{\max},\text{TEC}$	MILP (TI), CP
	[8]	$FJm\|\|C_{\max},\text{TEC}$	MILP (TI), H
flow-shop scheduling	[9]	$F2\|prmu, \text{on/off}\|\text{TEC}$	MILP (PI+TI), LBBD
parallel machine scheduling	[10]	$Pm\|\|C_{\max},\text{TEC}$	MILP (TI), H
	[11]	$Rm\|\|\text{TEC}$	MILP (PI), H
	[12]	$Rm\|\|\text{TEC}$	MILP (PI), B&P
single machine scheduling	[13]	$1\|batch\|\text{TEC}$	MILP (PI,TI), CG-H
	[14]	$1\|batch\|\text{TEC}$	MILP (PI)
RCPSP	[15]	$PS\|prec\|C_{\max},\text{TEC}$	MH
	[16]	$PS\|prec\|\text{TE}$	MILP (EB)

Table: An overview of related works.

*Graham's 3-field notation [3]

Short literature review

Problem class	Article	Problem*	Solution Approach
job-shop scheduling	*this work*	$Jm\|P_{max}\|TEC$	MILP (PI), B&C
	[4]	$Jm\|\|TEC$	MILP (PI)
	[5]	$Jm\|$ on/off, r_j, $d_j\|TEC$	MILP (TI), B&B
	[6]	$Jm\|P_{max}\|TEC$	MILP (D,TI), MH
flexible job-shop scheduling	[7]	$FJm\|$ on/off $\|C_{max}$,TEC	MILP (TI), CP
	[8]	$FJm\|\|C_{max}$,TEC	MILP (TI), H
flow-shop scheduling	[9]	$F2\|prmu$, on/off $\|TEC$	MILP (PI+TI), LBBD
parallel machine scheduling	[10]	$Pm\|\|C_{max}$,TEC	MILP (TI), H
	[11]	$Rm\|\|TEC$	MILP (PI), H
	[12]	$Rm\|\|TEC$	MILP (PI), B&P
single machine scheduling	[13]	$1\|batch\|TEC$	MILP (PI,TI), CG-H
	[14]	$1\|batch\|TEC$	MILP (PI)
RCPSP	[15]	$PS\|prec\|C_{max}$,TEC	MH
	[16]	$PS\|prec\|TE$	MILP (EB)

Table: An overview of related works.

*Graham's 3-field notation [3]

Main contributions

In this work, we introduce:

- a Branch-and-Cut algorithm for the $Jm|\mathrm{P}_{\mathrm{max}}|\mathrm{TEC}$ based on
  - » a period-indexed MILP formulation
  - » feasibility cuts generated from knapsack minimal covers
  - » a polynomial-time separation algorithm at integer infeasible nodes
- an extension of feasibility cuts
  - » from knapsack extended covers
  - » from any valid inequality of the capacity constraint polytope
- valid inequalities to improve the linear relaxation and the B&B tree exploration

Outline

1. Introduction
2. Branch-and-Cut approach
3. Computational results
4. Conclusion and perspectives

Outline

Variables

Processing status

$$x_{j,m}^{p} = \begin{cases} 1, & \text{if operation } (j,m) \text{ is processed during period } p \\ 0, & \text{otherwise} \end{cases} \qquad \forall j \in \mathcal{J}, m \in \mathcal{M}, p \in \mathcal{P}$$

Processing duration

$d_{j,m}^{p} \in \mathbb{R}^{+}$: time spent processing operation (j,m) on period p. $\qquad \forall j \in \mathcal{J}, m \in \mathcal{M}, p \in \mathcal{P}$

Variables

Processing status

$$x^p_{j,m} = \begin{cases} 1, & \text{if operation } (j,m) \text{ is processed during period } p \\ 0, & \text{otherwise} \end{cases} \qquad \forall j \in \mathcal{J}, m \in \mathcal{M}, p \in \mathcal{P}$$

Processing duration

$d^p_{j,m} \in \mathbb{R}^+$: time spent processing operation (j,m) on period p. $\quad \forall j \in \mathcal{J}, m \in \mathcal{M}, p \in \mathcal{P}$

Starting/completion date

$s_{j,m}, c_{j,m} \in \mathbb{R}^+$: starting and completion dates of operation (j,m) $\quad \forall j \in \mathcal{J}, m \in \mathcal{M}$

Machine disjunction

$$u^{j,m}_{j',m} = \begin{cases} 1, & \text{processing of operation } (j,m) \text{ ends before start of } (j',m') \\ 0, & \text{otherwise} \end{cases} \qquad \forall j < j' \in \mathcal{J}, m, m' \in \mathcal{M}$$

Objective function

Schedule total cost

The total operational cost of a schedule is minimized:

$$\min \sum_{p \in \mathcal{P}} c^p \sum_{m \in M} \varphi_m \sum_{j \in \mathcal{J}} d^p_{j,m} \tag{1}$$

- φ_m: power of machine m,
- c^p: cost of period p.

Core constraints

Total operation processing

$$\sum_{p \in \mathcal{P}} d^{p}_{j,m} = q_{j,m}, \quad \forall (j, m) \in \mathcal{O}. \tag{2}$$

$q_{j,m}$: duration of (j, m)

Core constraints

Total operation processing

$$\sum_{p \in \mathcal{P}} d^p_{j,m} = q_{j,m}, \quad \forall (j,m) \in \mathcal{O}. \tag{2}$$

$q_{j,m}$: duration of (j, m)

Machine disjunction

$$\text{(def.)} \qquad c_{j,m} - s_{j',m'} \leqslant \alpha^{j',m'}_{j,m}(1 - u^{j',m'}_{j,m}), \qquad \forall (j,m),(j',m') \in \mathcal{O} : j \neq j' \tag{3a}$$

$$u^{j',m}_{j,m} + u^{j,m}_{j',m} = 1, \qquad \forall m \in \mathcal{M}, \forall j,j' \in \mathcal{J} : j < j' \tag{3b}$$

$\alpha_{j,j',m}$: constant

Core constraints

Total operation processing

$$\sum_{p \in \mathcal{P}} d_{j,m}^{p} = q_{j,m}, \quad \forall (j,m) \in \mathcal{O}. \tag{2}$$

$q_{j,m}$: duration of (j,m)

Machine disjunction

$$\text{(def.)} \qquad c_{j,m} - s_{j',m'} \leqslant \alpha_{j,m}^{j',m'} (1 - u_{j,m}^{j',m'}), \qquad \forall (j,m),(j',m') \in \mathcal{O} : j \neq j' \tag{3a}$$

$$u_{j,m}^{j',m} + u_{j',m}^{j,m} = 1, \qquad \forall m \in \mathcal{M}, \forall j,j' \in \mathcal{J} : j < j' \tag{3b}$$

$\alpha_{j,j',m}$: constant

Precedence

$$c_{j,m} \leqslant s_{j,m'}, \quad \forall j \in \mathcal{J}, \forall m,m' \in \mathcal{M} : (j,m) \prec (j,m'). \tag{4}$$

Variable linking

Variable linking: x and d

$$d^{p}_{j,m} \leqslant \min\{l^{p}, \overline{d}^{p}_{j,m}, q_{j,m}\} \cdot x^{p}_{j,m}, \quad \forall (j,m) \in \mathcal{O}, \forall p \in \mathcal{P}. \tag{5}$$

---

l^p: length of period p

Variable linking

Variable linking: x and d

$$d^p_{j,m} \leqslant \min\{l^p, \overline{d}^p_{j,m}, q_{j,m}\} \cdot x^p_{j,m}, \quad \forall (j,m) \in \mathcal{O}, \forall p \in \mathcal{P}. \tag{5}$$

l^p: length of period p

Variable linking: x, d and s

$$d^p_{j,m} \leqslant t^{p+1} - s_{j,m} + \gamma^p_{j,m} \cdot (1 - x^p_{j,m}), \quad \forall (j,m) \in \mathcal{O}, \forall p \in \mathcal{P}, \tag{6a}$$

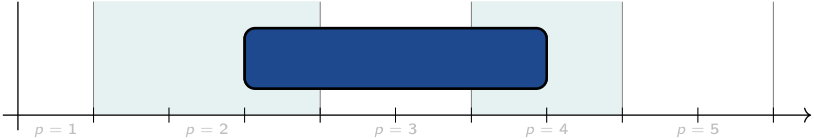
$$d^p_{j,m} \leqslant c_{j,m} - t^p \cdot x^p_{j,m}, \quad \forall (j,m) \in \mathcal{O}, \forall p \in \mathcal{P}. \tag{6b}$$

These constraints guarantee **non-preemption**.

$[t^p, t^{p+1}]$: period p

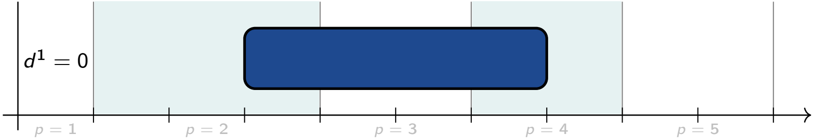
$\gamma^p_{j,m}$: constant

Variable linking: numerical example



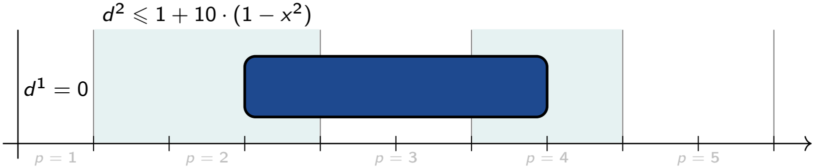
Variable linking: numerical example

- For p = 1, (6a) $\Rightarrow x^1 = 0 \Rightarrow d^1 = 0$,



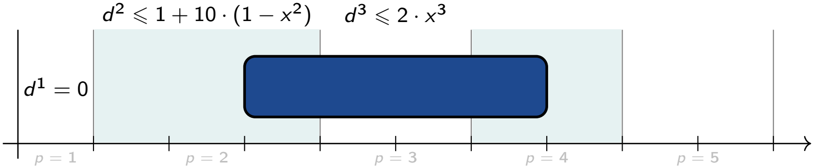
Variable linking: numerical example

- For p = 1, (6a) $\Rightarrow x^1 = 0 \Rightarrow d^1 = 0$,
- For p = 2, (6a) $\Rightarrow d^2 \leqslant 1 + 10 \cdot (1 - x^2)$,



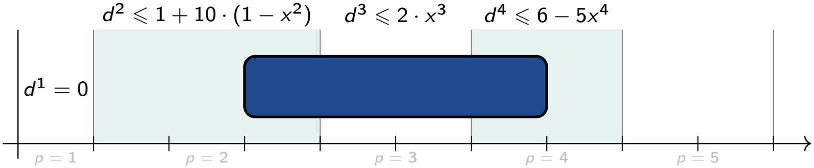
Variable linking: numerical example

- For p = 1, (6a) $\Rightarrow x^1 = 0 \Rightarrow d^1 = 0$,
- For p = 2, (6a) $\Rightarrow d^2 \leqslant 1 + 10 \cdot (1 - x^2)$,
- For p = 3, (5) $\Rightarrow d^3 \leqslant 2 \cdot x^3$



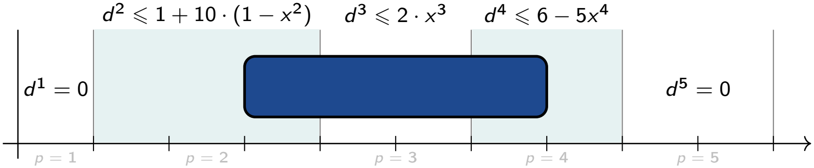
Variable linking: numerical example

- For p = 1, (6a) $\Rightarrow x^1 = 0 \Rightarrow d^1 = 0$,
- For p = 2, (6a) $\Rightarrow d^2 \leqslant 1 + 10 \cdot (1 - x^2)$,
- For p = 3, (5) $\Rightarrow d^3 \leqslant 2 \cdot x^3$
- For p = 4, (6b) $\Rightarrow d^4 \leqslant 6 - 5x^4$,



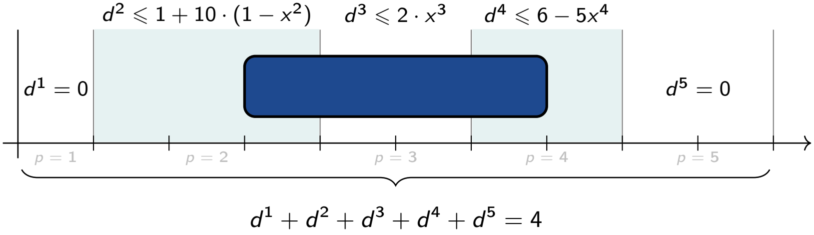
Variable linking: numerical example

- For p = 1, (6a) $\Rightarrow x^1 = 0 \Rightarrow d^1 = 0$,
- For p = 2, (6a) $\Rightarrow d^2 \leqslant 1 + 10 \cdot (1 - x^2)$,
- For p = 3, (5) $\Rightarrow d^3 \leqslant 2 \cdot x^3$
- For p = 4, (6b) $\Rightarrow d^4 \leqslant 6 - 5x^4$,
- For p = 5, (6b) $\Rightarrow x^5 = 0 \Rightarrow d^5 = 0$,



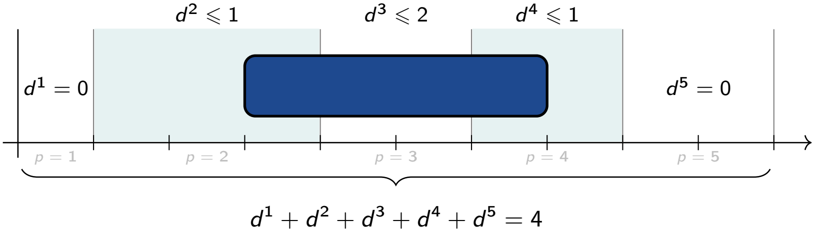
Variable linking: numerical example

- For p = 1, (6a) $\Rightarrow x^1 = 0 \Rightarrow d^1 = 0$,
- For p = 2, (6a) $\Rightarrow d^2 \leqslant 1 + 10 \cdot (1 - x^2)$,
- For p = 3, (5) $\Rightarrow d^3 \leqslant 2 \cdot x^3$
- For p = 4, (6b) $\Rightarrow d^4 \leqslant 6 - 5x^4$,
- For p = 5, (6b) $\Rightarrow x^5 = 0 \Rightarrow d^5 = 0$,
- (2) $\Rightarrow \sum_{p \in \mathcal{P}} d^p = q$



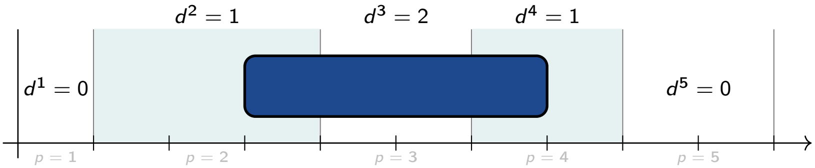
Variable linking: numerical example

- For p = 1, (6a) $\Rightarrow x^1 = 0 \Rightarrow d^1 = 0$,
- For p = 2, (6a) $\Rightarrow d^2 \leqslant 1 + 10 \cdot (1 - x^2)$,
- For p = 3, (5) $\Rightarrow d^3 \leqslant 2 \cdot x^3$
- For p = 4, (6b) $\Rightarrow d^4 \leqslant 6 - 5x^4$,
- For p = 5, (6b) $\Rightarrow x^5 = 0 \Rightarrow d^5 = 0$,
- (2) $\Rightarrow \sum_{p \in \mathcal{P}} d^p = q$



Variable linking: numerical example

- For p = 1, (6a) $\Rightarrow x^1 = 0 \Rightarrow d^1 = 0$,
- For p = 2, (6a) $\Rightarrow d^2 \leqslant 1 + 10 \cdot (1 - x^2)$,
- For p = 3, (5) $\Rightarrow d^3 \leqslant 2 \cdot x^3$
- For p = 4, (6b) $\Rightarrow d^4 \leqslant 6 - 5x^4$,
- For p = 5, (6b) $\Rightarrow x^5 = 0 \Rightarrow d^5 = 0$,
- (2) $\Rightarrow \sum_{p \in \mathcal{P}} d^p = q$



Capacity constraint

- $z_{j,m}^{j',m'} := 1 - u_{j,m}^{j',m'} - u_{j',m'}^{j,m}$ denotes overlap of (j, m) and (j', m').
- $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ weighted graph induced by z
  - » $\mathcal{V}$: operations (j, m) with weights φ_m
  - » $\mathcal{E}$: operations overlap status
- Overlapping operations form a clique K in $\mathcal{G}$, of weight $\varphi[K] := \sum_{m \in K} \varphi_m$.
- Total power demand $\varphi_{\text{tot}} > \overline{\varphi} \iff$ Clique weight $\varphi[K] > \overline{\varphi}$. $\quad (\exists K \subseteq \mathcal{G})$

Capacity constraint

- $z_{j,m}^{j',m'} := 1 - u_{j,m}^{j',m'} - u_{j',m'}^{j,m}$ denotes overlap of (j, m) and (j', m').
- $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ weighted graph induced by z
  - $\mathcal{V}$: operations (j, m) with weights φ_m
  - $\mathcal{E}$: operations overlap status
- Overlapping operations form a clique K in $\mathcal{G}$, of weight $\varphi[K] := \sum_{m \in K} \varphi_m$.
- Total power demand $\varphi_{\text{tot}} > \overline{\varphi} \iff$ Clique weight $\varphi[K] > \overline{\varphi}$. $\quad (\exists K \subseteq \mathcal{G})$

Capacity

Forbid violating cliques:

$$\sum_{e \in E(K)} z^e \leqslant |E(K)| - 1, \quad \forall K \subseteq \mathcal{G} : \varphi[K] > \overline{\varphi} \tag{7}$$

Capacity constraint

- Better: **forbid minimally violating cliques**.
- Capacity constraint *Knapsack set* $\mathrm{KP}^{\varphi,\overline{\varphi}} := \left\{x \in \{0,1\}^n : \sum_{j=1}^{n} \varphi_j x_j \leqslant \overline{\varphi}\right\}$.
- $C \subseteq \{1, \cdots, n\}$ is a *minimal cover* of $\mathrm{KP}^{\varphi,\overline{\varphi}}$ if
  - » $\sum_{i \in C} \varphi_i > \overline{\varphi}$,
  - » $\sum_{i \in C \setminus \{j\}} \varphi_i \leqslant \overline{\varphi}, \quad \forall j \in C$.

→ If K_C induced by minimal cover $C \in \mathcal{C}$, then any $H \subsetneq K_C$ satisfies $\varphi[H] \leqslant \overline{\varphi}$.

Capacity constraint

- Better: **forbid minimally violating cliques.**
- Capacity constraint *Knapsack set* $\mathrm{KP}^{\varphi,\overline{\varphi}} := \left\{x \in \{0,1\}^n : \sum_{j=1}^n \varphi_j x_j \leqslant \overline{\varphi}\right\}$.
- $C \subseteq \{1, \cdots, n\}$ is a *minimal cover* of $\mathrm{KP}^{\varphi,\overline{\varphi}}$ if
  - » $\sum_{i \in C} \varphi_i > \overline{\varphi}$,
  - » $\sum_{i \in C \setminus \{j\}} \varphi_i \leqslant \overline{\varphi}, \quad \forall j \in C$.

→ If K_C induced by minimal cover $C \in \mathcal{C}$, then any $H \subsetneq K_C$ satisfies $\varphi[H] \leqslant \overline{\varphi}$.

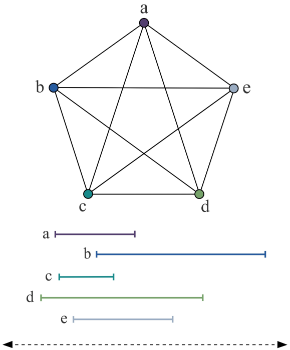
Capacity

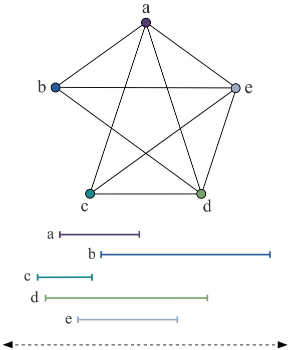
Forbid minimally violating cliques:

$$\sum_{e \in E(K_C)} z^e \leqslant |E(K_C)| - 1, \quad \forall C \in \mathcal{C}, \forall K_C \subseteq \mathcal{G} \tag{8}$$

Capacity constraint: numerical example

- 5 overlapping operations (a,b,c,d,e) s.t. $\varphi = [1, 3, 3, 4, 5]$ and $\overline{\varphi} = 15$.
  - » $\{a, b, c, d, e\}$ is a minimal cover,





  - » (7) $\Rightarrow \sum_{e \in E(K)} z^e \leqslant |E(K)| - 1$

$$\Rightarrow z_a^b + z_a^c + z_a^d + z_a^e + z_b^c + z_b^d + z_b^e + z_c^d + z_c^e + z_d^e \leqslant 9$$

Capacity constraint

- Better: **forbid minimally violating cliques.**
- Capacity constraint *Knapsack set* $\mathrm{KP}^{\varphi,\overline{\varphi}} := \{x \in \{0,1\}^n : \sum_{j=1}^{n} \varphi_j x_j \leqslant \overline{\varphi}\}$.
- $C \subseteq \{1, \cdots, n\}$ is a *minimal cover* of $\mathrm{KP}^{\varphi,\overline{\varphi}}$ if

Exponentially many constraints

For each minimal cover, forbid all assignments of jobs to machines in cover, i.e. permutations

» Exponentially growing number of constraints $\sum_{C \in \mathcal{C}} \frac{|\mathcal{J}|!}{(|\mathcal{J}|-|C|)!}$

Capacity

Forbid minimally violating cliques:

$$\sum_{e \in E(K_C)} z^e \leqslant |E(K_C)| - 1, \quad \forall C \in \mathcal{C}, \forall K_C \subseteq \mathcal{G} \tag{9}$$

Capacity constraint

- Better: **forbid minimally violating cliques.**
- Capacity constraint *Knapsack set* $\mathrm{KP}^{\varphi,\overline{\varphi}} := \{x \in \{0,1\}^n : \sum_{j=1}^n \varphi_j x_j \leqslant \overline{\varphi}\}$.
- $C \subseteq \{1, \cdots, n\}$ is a *minimal cover* of $\mathrm{KP}^{\varphi,\overline{\varphi}}$ if

Exponentially many constraints

For each minimal cover, forbid all assignments of jobs to machines in cover, i.e. permutations

» Exponentially growing number of constraints $\sum_{C \in \mathcal{C}} \frac{|\mathcal{J}|!}{(|\mathcal{J}|-|C|)!}$ → **dynamic separation**

Capacity

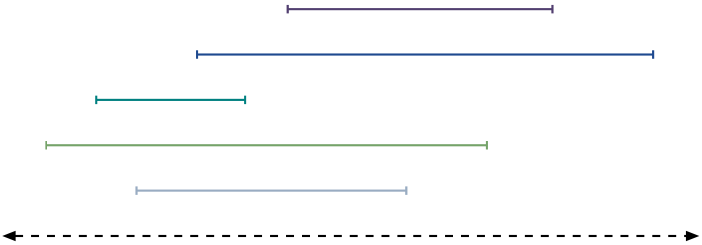
Forbid minimally violating cliques:

$$\sum_{e \in E(K_C)} z^e \leqslant |E(K_C)| - 1, \quad \forall C \in \mathcal{C}, \forall K_C \subseteq \mathcal{G} \tag{9}$$

Outline

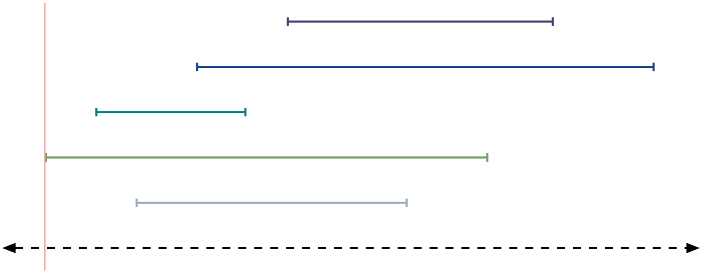
Capacity constraint: separation

- At integer infeasible nodes
- Sufficient condition: no violated inequalities from minimal covers
- Scan the schedule at event middle-points
  - » Event: start or completion of an operation
  - » Power demand is constant between consecutive events, (e.g.) at middle



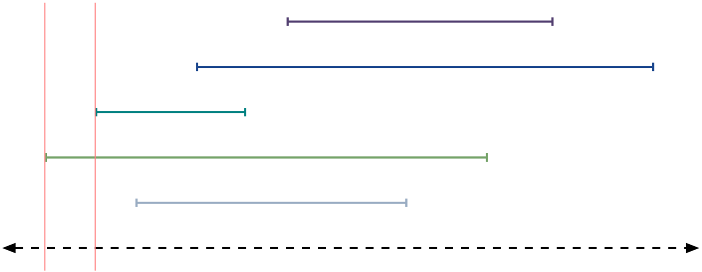
Capacity constraint: separation

- At integer infeasible nodes
- Sufficient condition: no violated inequalities from minimal covers
- Scan the schedule at event middle-points
  - » Event: start or completion of an operation
  - » Power demand is constant between consecutive events, (e.g.) at middle



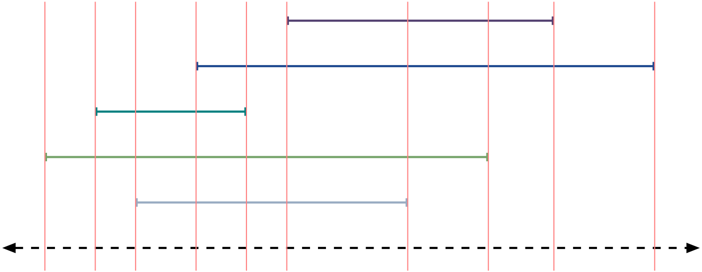
Capacity constraint: separation

- At integer infeasible nodes
- Sufficient condition: no violated inequalities from minimal covers
- Scan the schedule at event middle-points
  - Event: start or completion of an operation
  - Power demand is constant between consecutive events, (e.g.) at middle



Capacity constraint: separation

- At integer infeasible nodes
- Sufficient condition: no violated inequalities from minimal covers
- Scan the schedule at event middle-points
  - » Event: start or completion of an operation
  - » Power demand is constant between consecutive events, (e.g.) at middle



Outline

Knapsack extended covers

- For a minimal cover C, an extended cover is

$$E(C) := C \cup \{j \in \mathcal{M} \mid \varphi_j \geqslant \varphi_i, \forall i \in C\}$$

- If $\{j \in \mathcal{M} \mid \varphi_j \geqslant \varphi_i, \forall i \in C\}$ non-empty $\Rightarrow$ extended cover inequality

$$\sum_{j \in E(C)} x_j \leqslant |C| - 1$$

  dominates minimal cover inequality.

- $n := |E(C)|, r := |C| - 1$: pick at most r among the n items.

Clique-forbidding cuts from any cover

- Let $n := |E(C)|, r := |C| - 1$: with $n, r \in \mathbb{N}^*$ and $n > r$. ‡
  - In scheduling terms: among n machines, at most r can execute simultaneously.
  - In graph theory terms: among n-vertex graphs, allow at most K_r, i.e. forbid K_{r+1}.
- Idea: extremal graphs
  - *r-extremal* graph: maximizes the number of edges among K_{r+1}-free graphs.
  - $\mathtt{ex}(n, K_r)$: edge count of the r-extremal n-vertex graph $\rightarrow$ valid UB

$$\sum_{e \in E(K_n)} z^e \leqslant \mathtt{ex}(n, K_r), \quad \forall K_n \subseteq \mathcal{G} : K_{r+1}\text{-free} \tag{10}$$

  - $\mathtt{ex}(n, K_r)$ is bounded (from above) by $(1 - \frac{1}{r})\frac{n^2}{2}$ [17].

‡ $n = r + 1$ for minimal covers

Clique-forbidding cuts from any cover

- The overlap graph $\mathcal{G}$, induced by z, is an *interval graph*
  - » $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is an *interval graph* if each $v \in \mathcal{V}$ can be assigned an interval on the real line, s.t. two intervals intersect iff the corresponding vertices are adjacent.
- In that case, we obtain a tighter UB:

Extremal interval graphs

The edge count in an n-vertex r-extremal interval graph is

$$\mathrm{ex}(n, K_r) = \binom{n-1}{2} - \binom{n-r+1}{2}. \tag{11}$$

Proof idea: rearrange intervals in $\nearrow$ of right end-point and reason on complement graph.

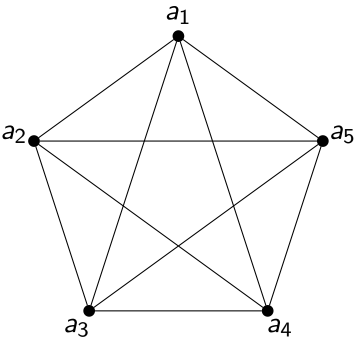
- Given in Abbott and Katchalski [18] (1979) as $\mathrm{ex}(n, K_r) = \binom{r}{2} + (n-r)(r-1)$.

Clique-forbidding cuts from any valid inequality

- Consider $a^{\intercal}x \leqslant b$ valid for $\text{conv}\{KP^{\varphi,\overline{\varphi}}\}$. $a \in \mathbb{R}_n^+, b \in \mathbb{R}^+$.

  $\rightarrow$ Find a corresponding inequality $\sum_{e \in E(K_n)} \alpha^e z^e \leqslant \beta$

- Two cases
  - » $a \in \{0,1\}^n$
  - » $a \notin \{0,1\}^n$

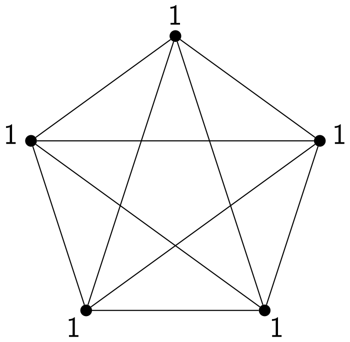


Clique-forbidding cuts from any valid inequality

- Consider $a^{\intercal}x \leqslant b$ valid for $\mathrm{conv}\{\mathrm{KP}^{\varphi,\overline{\varphi}}\}$. $a \in \mathbb{R}_n^+, b \in \mathbb{R}^+$.

  $\rightarrow$ Find a corresponding inequality $\sum_{e \in E(K_n)} \alpha^e z^e \leqslant \beta$

- Two cases
  - » $a \in \{0,1\}^n \Rightarrow \sum_{e \in \mathcal{E}} z^e \leqslant \mathrm{ex}(n, K_{b+1})$
  - » $a \notin \{0,1\}^n$

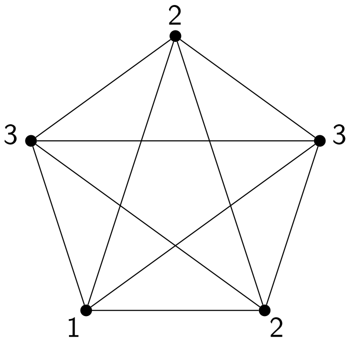


Clique-forbidding cuts from any valid inequality

- Consider $a^\intercal x \leqslant b$ valid for $\text{conv}\{KP^{\varphi,\overline{\varphi}}\}$. $a \in \mathbb{R}_n^+, b \in \mathbb{R}^+$.

  $\rightarrow$ Find a corresponding inequality $\sum_{e \in E(K_n)} \alpha^e z^e \leqslant \beta$

- Two cases
  - » $a \in \{0,1\}^n \Rightarrow \sum_{e \in \mathcal{E}} z^e \leqslant \text{ex}(n, K_{b+1})$
  - » $a \notin \{0,1\}^n \Rightarrow$?

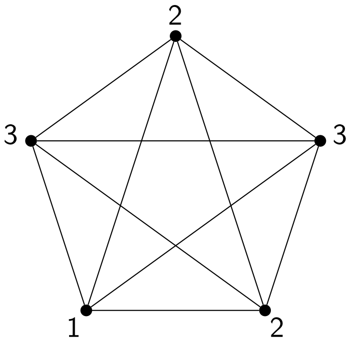


Clique-forbidding cuts from any valid inequality

- Consider $a^{\intercal}x \leqslant b$ valid for $\text{conv}\{\text{KP}^{\varphi,\overline{\varphi}}\}$. $a \in \mathbb{R}_n^+, b \in \mathbb{R}^+$.

  $\rightarrow$ Find a corresponding inequality $\sum_{e \in E(K_n)} \alpha^e z^e \leqslant \beta$

- Idea : $n = \underbrace{1 + \ldots + 1}_{n \text{ times}}$



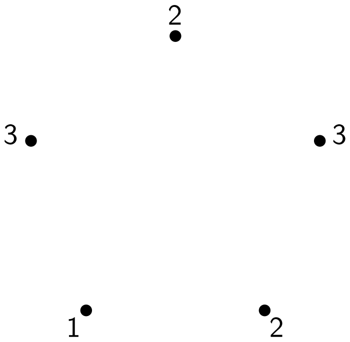
Clique-forbidding cuts from any valid inequality

- Consider $a^\intercal x \leqslant b$ valid for $\text{conv}\{KP^{\varphi,\overline{\varphi}}\}$. $a \in \mathbb{R}^+_n$, $b \in \mathbb{R}^+$.

  $\rightarrow$ Find a corresponding inequality $\sum_{e \in E(K_n)} \alpha^e z^e \leqslant \beta$

- Idea : $n = \underbrace{1 + \ldots + 1}_{n \text{ times}}$

  i) Duplicate each vertex as many times as its weight



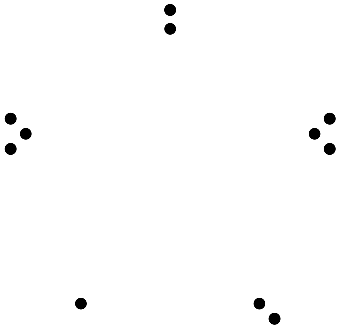
Clique-forbidding cuts from any valid inequality

- Consider $a^\intercal x \leqslant b$ valid for $\text{conv}\{\text{KP}^{\varphi,\overline{\varphi}}\}$. $a \in \mathbb{R}^+_n, b \in \mathbb{R}^+$.

  $\rightarrow$ Find a corresponding inequality $\sum_{e \in E(K_n)} \alpha^e z^e \leqslant \beta$

- Idea : $n = \underbrace{1 + \ldots + 1}_{n \text{ times}}$

  i) Duplicate vertices $\Rightarrow$ $\sum_i a_i$ vertices



Clique-forbidding cuts from any valid inequality

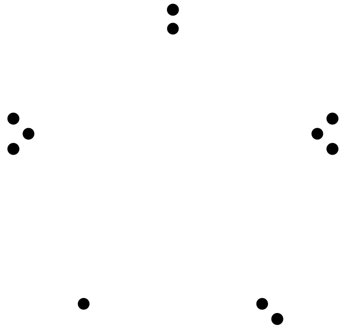
- Consider $a^\intercal x \leqslant b$ valid for $\text{conv}\{KP^{\varphi,\overline{\varphi}}\}$. $a \in \mathbb{R}_n^+, b \in \mathbb{R}^+$.

  $\rightarrow$ Find a corresponding inequality $\sum_{e \in E(K_n)} \alpha^e z^e \leqslant \beta$

- Idea : $n = \underbrace{1 + \ldots + 1}_{n \text{ times}}$

  i) Duplicate vertices $\Rightarrow$ $\sum_i a_i$ vertices

  ii) Take the (unweighted) complete graph on the resulting set of vertices



Clique-forbidding cuts from any valid inequality

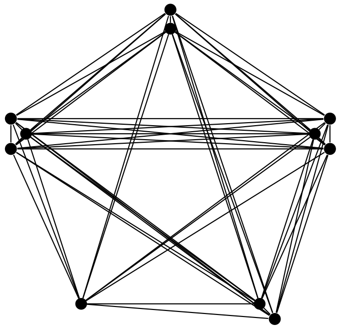
- Consider $a^\intercal x \leqslant b$ valid for $\text{conv}\{KP^{\varphi,\overline{\varphi}}\}$. $a \in \mathbb{R}^+_n, b \in \mathbb{R}^+$.

  $\rightarrow$ Find a corresponding inequality $\sum_{e \in E(K_n)} \alpha^e z^e \leqslant \beta$

- Idea : $n = \underbrace{1 + \ldots + 1}_{n \text{ times}}$

  i) Duplicate vertices $\Rightarrow \sum_i a_i$ vertices

  ii) Complete graph $\Rightarrow \binom{\sum_i a_i}{2}$ edges



Clique-forbidding cuts from any valid inequality

- Consider $a^\intercal x \leqslant b$ valid for $\text{conv}\{KP^{\varphi,\overline{\varphi}}\}$. $a \in \mathbb{R}_n^+, b \in \mathbb{R}^+$.

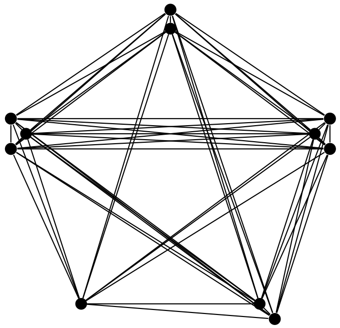
  $\rightarrow$ Find a corresponding inequality $\sum_{e \in E(K_n)} \alpha^e z^e \leqslant \beta$

- Idea : $n = \underbrace{1 + \ldots + 1}_{n \text{ times}}$

  i) Duplicate vertices $\Rightarrow \sum_i a_i$ vertices

  ii) Complete graph $\Rightarrow \binom{\sum_i a_i}{2}$ edges

  iii) Forbid K_{b+1}: apply previous result on extended covers



Clique-forbidding cuts from any valid inequality

- Consider $a^\intercal x \leqslant b$ valid for $\text{conv}\{KP^{\varphi,\overline{\varphi}}\}$. $a \in \mathbb{R}^+_n, b \in \mathbb{R}^+$.

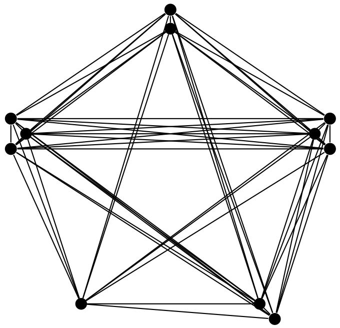
  $\rightarrow$ Find a corresponding inequality $\sum_{e \in E(K_n)} \alpha^e z^e \leqslant \beta$

- Idea : $n = \underbrace{1 + \ldots + 1}_{n \text{ times}}$

  i) Duplicate vertices $\Rightarrow \sum_i a_i$ vertices

  ii) Complete graph $\Rightarrow \binom{\sum_i a_i}{2}$ edges

  iii) Forbid $K_{b+1} \Rightarrow \beta = \texttt{ex}(\binom{\sum_i a_i}{2}, K_{b+1})$



Clique-forbidding cuts from any valid inequality

- Consider $a^\intercal x \leqslant b$ valid for $\text{conv}\{\text{KP}^{\varphi,\overline{\varphi}}\}$. $a \in \mathbb{R}^+_n, b \in \mathbb{R}^+$.

  $\rightarrow$ Find a corresponding inequality $\sum_{e \in E(K_n)} \alpha^e z^e \leqslant \beta$

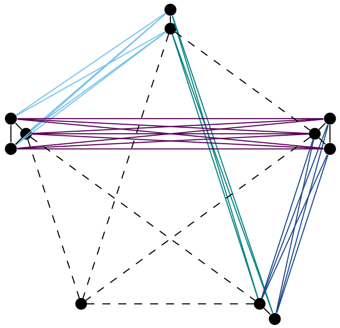
- Idea : $n = \underbrace{1 + \ldots + 1}_{n \text{ times}}$

  i) Duplicate vertices $\Rightarrow$ $\sum_i a_i$ vertices

  ii) Complete graph $\Rightarrow$ $\binom{\sum_i a_i}{2}$ edges

  iii) Forbid K_{b+1} $\Rightarrow$ $\beta = \texttt{ex}(\binom{\sum_i a_i}{2}, K_{b+1})$

  iv) Merge redundant edges in inequality



Clique-forbidding cuts from any valid inequality

- Consider $a^{\intercal}x \leqslant b$ valid for $\mathrm{conv}\{KP^{\varphi,\overline{\varphi}}\}$. $a \in \mathbb{R}_n^+, b \in \mathbb{R}^+$.

  $\rightarrow$ Find a corresponding inequality $\sum_{e \in E(K_n)} \alpha^e z^e \leqslant \beta$

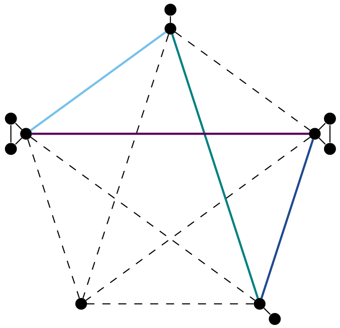
- Idea : $n = \underbrace{1 + \ldots + 1}_{n \text{ times}}$

  i) Duplicate vertices $\Rightarrow \sum_i a_i$ vertices

  ii) Complete graph $\Rightarrow \binom{\sum_i a_i}{2}$ edges

  iii) Forbid $K_{b+1} \Rightarrow \beta = \mathtt{ex}(\binom{\sum_i a_i}{2}, K_{b+1})$

  iv) Merge redundant $\Rightarrow \alpha^e = a_u a_v$ for any $e = (u, v)$



Clique-forbidding cuts from any valid inequality

- Consider $a^\intercal x \leqslant b$ valid for $\text{conv}\{\text{KP}^{\varphi,\overline{\varphi}}\}$. $a \in \mathbb{R}_n^+, b \in \mathbb{R}^+$.

  $\rightarrow$ Find a corresponding inequality $\sum_{e \in E(K_n)} \alpha^e z^e \leqslant \beta$

- Idea : $n = \underbrace{1 + \ldots + 1}_{n \text{ times}}$

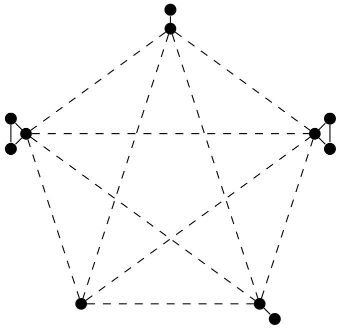
  i) Duplicate vertices $\Rightarrow \sum_i a_i$ vertices

  ii) Complete graph $\Rightarrow \binom{\sum_i a_i}{2}$ edges

  iii) Forbid $K_{b+1} \Rightarrow \beta = \text{ex}(\binom{\sum_i a_i}{2}, K_{b+1})$

  iv) Merge redundant $\Rightarrow \alpha^e = a_u a_v$ for any $e = (u, v)$

  v) Account for edges between vertex and its duplicates



Clique-forbidding cuts from any valid inequality

- Consider $a^\intercal x \leqslant b$ valid for $\text{conv}\{KP^{\varphi,\overline{\varphi}}\}$. $a \in \mathbb{R}^+_n$, $b \in \mathbb{R}^+$.

  $\rightarrow$ Find a corresponding inequality $\sum_{e \in E(K_n)} \alpha^e z^e \leqslant \beta$

- Idea : $n = \underbrace{1 + \ldots + 1}_{n \text{ times}}$

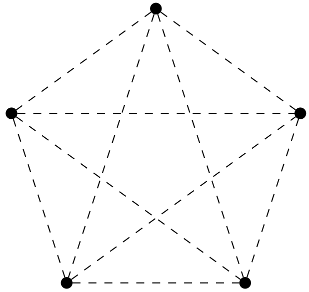
  i) Duplicate vertices $\Rightarrow \sum_i a_i$ vertices

  ii) Complete graph $\Rightarrow \binom{\sum_i a_i}{2}$ edges

  iii) Forbid $K_{b+1} \Rightarrow \beta = \text{ex}(\binom{\sum_i a_i}{2}, K_{b+1})$

  iv) Merge redundant $\Rightarrow \alpha^e = a_u a_v$ for any $e = (u, v)$

  v) Count dups $\Rightarrow \beta = \text{ex}\left(\binom{\sum_i a_i}{2}, K_{b+1}\right) - \sum_{a_u \geqslant 2} \binom{a_u}{2}$



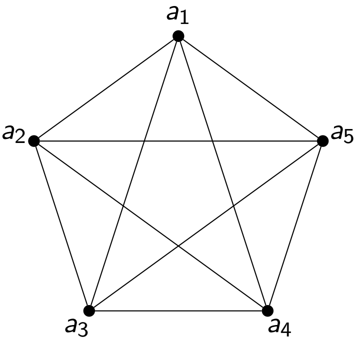
Clique-forbidding cuts from any valid inequality

- Consider $a^{\intercal}x \leqslant b$ valid for $\text{conv}\{\text{KP}^{\varphi,\overline{\varphi}}\}$. $a \in \mathbb{R}^+_n$, $b \in \mathbb{R}^+$.

  $\rightarrow$ Find a corresponding inequality $\sum_{e \in E(K_n)} \alpha^e z^e \leqslant \beta$

- $$\begin{cases} \alpha^e = a_u a_v, & \text{for } e = (u, v) \\ \beta = \text{ex}\left(\binom{\sum_i a_i}{2}, K_{b+1}\right) - \sum_{u \in \mathcal{V}: a_u \geqslant 2} \binom{a_u}{2} \end{cases}$$

- Unified formula.



⚠ $\sum_{e \in E(K_n)} \alpha^e z^e \leqslant \beta \Rightarrow a^{\intercal}x \leqslant b$ does **not always** hold.

Clique-forbidding cuts from any valid inequality: example

- Given $\mathrm{KP}^{\varphi,\overline{\varphi}}$ where $[\varphi] = [1, 3, 3, 4, 5]$ and $\overline{\varphi} = 10$.
  - » Minimal cover: $\{1, 2, 3, 4\} \Rightarrow \sum_{e \in E(K_{\{1,2,3,4\}})} z^e \leqslant 3$.
  - » Extended cover: $\{1, 2, 3, 4, 5\} \Rightarrow \sum_{e \in E(K_{\{1,2,3,4,5\}})} z^e \leqslant 7$.
  - » Lifted cover: $a = [0, 1, 1, 1, 2]$ and $b = 3 \Rightarrow \sum_{(u,v) \in E(K_{\{2,3,4,5\}})} a_u a_v z^{(u,v)} \leqslant 6$

    ⚠ If $\mathrm{x} = [0, 0, 1, 1, 1]$, then $a^\intercal \mathrm{x} = 4 > b = 3$,

    but $\sum_{e \in E(K_{\{2,3,4,5\}})} \alpha^e z^e = 5 \leqslant 6$.

A period-indexed MILP formulation

$$
\begin{aligned}
\min \quad & \sum_{p\in\mathcal{P}} c^p \sum_{m\in M} \varphi_m \sum_{j\in\mathcal{J}} d^p_{j,m}, && && \text{(12a)}\\
[s.t.] \quad & \sum_{p\in\mathcal{P}} d^p_{j,m} = q_{j,m}, && \forall (j,m)\in\mathcal{O}, && \text{(12b)}\\
& d^p_{j,m} \leqslant \min\{l^p, \overline{d}^p_{j,m} q_{j,m}\}\cdot x^p_{j,m}, && \forall (j,m)\in\mathcal{O}, \forall p\in\mathcal{P}, && \text{(12c)}\\
& c_{j,m} - s_{j',m'} \leqslant \alpha^{j',m'}_{j,m}(1-u^{j',m'}_{j,m}), && \forall (j,m),(j',m')\in\mathcal{O} : j\neq j' && \text{(12d)}\\
& u^{j',m'}_{j,m} + u^{j,m}_{j',m} = 1, && \forall m\in\mathcal{M}, \forall j,j'\in\mathcal{J} : j<j' && \text{(12e)}\\
& c_{j,m} \leqslant s_{j,m'}, && \forall j\in\mathcal{J}, \forall m,m'\in\mathcal{M} : (j,m)\prec(j,m') && \text{(12f)}\\
& d^p_{j,m} \leqslant t^{p+1} - s_{j,m} + \gamma^p_{j,m}\cdot(1-x^p_{j,m}), && \forall (j,m)\in\mathcal{O}, \forall p\in\mathcal{P}, && \text{(12g)}\\
& d^p_{j,m} \leqslant c_{j,m} - t^p\cdot x^p_{j,m}, && \forall (j,m)\in\mathcal{O}, \forall p\in\mathcal{P}, && \text{(12h)}\\
& \sum_{e\in E(K_C)} z^e \leqslant |E(K_C)| - 1, && \forall C\in\mathcal{C}, \forall K_C\subseteq\mathcal{G}, && \text{(12i)}\\
& x^p_{j,m}\in\{0,1\}, d^p_{j,m}\geqslant 0, && \forall (j,m)\in\mathcal{O}, \forall p\in\mathcal{P}, && \text{(12j)}\\
& u^{j',m'}_{j,m}\in\{0,1\}, s_{j,m}\geqslant 0, && \forall j,j'\in\mathcal{J} : j<j', \forall m,m'\in\mathcal{M}. && \text{(12k)}
\end{aligned}
$$

A period-indexed MILP formulation

$$\min \quad \sum_{p\in\mathcal{P}} c^p \sum_{m\in M} \varphi_m \sum_{j\in\mathcal{J}} d^p_{j,m}, \tag{12a}$$

$$[s.t.] \quad \sum_{p\in\mathcal{P}} d^p_{j,m} = q_{j,m}, \qquad \forall (j,m)\in\mathcal{O}, \tag{12b}$$

$$d^p_{j,m} \leqslant \min\{l^p, \overline{d}^p_{j,m} q_{j,m}\} \cdot x^p_{j,m}, \qquad \forall (j,m)\in\mathcal{O}, \forall p\in\mathcal{P}, \tag{12c}$$

Formulation properties

Natural date + disjunction variables / big-M formulation ⇒ weak LP relaxations (see e.g. [19]).

$$d^p_{j,m} \leqslant t^{p+1} - s_{j,m} + \gamma^p_{j,m} \cdot (1 - x^p_{j,m}), \qquad \forall (j,m)\in\mathcal{O}, \forall p\in\mathcal{P}, \tag{12g}$$

$$d^p_{j,m} \leqslant c_{j,m} - t^p \cdot x^p_{j,m}, \qquad \forall (j,m)\in\mathcal{O}, \forall p\in\mathcal{P}, \tag{12h}$$

$$\sum_{e\in E(K_C)} z^e \leqslant |E(K_C)| - 1, \qquad \forall C\in\mathcal{C}, \forall K_C \subseteq \mathcal{G}, \tag{12i}$$

$$x^p_{j,m} \in \{0,1\}, d^p_{j,m} \geqslant 0, \qquad \forall (j,m)\in\mathcal{O}, \forall p\in\mathcal{P}, \tag{12j}$$

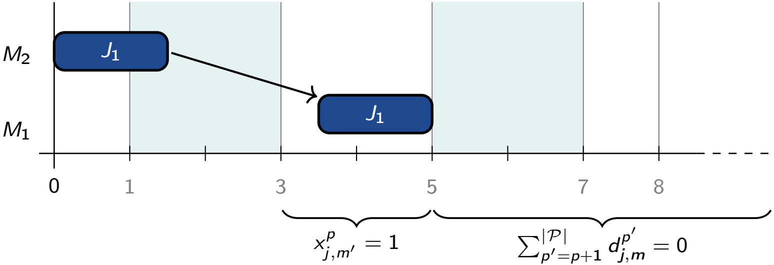
$$u^{j',m'}_{j,m} \in \{0,1\}, s_{j,m} \geqslant 0, \qquad \forall j,j'\in\mathcal{J} : j < j', \forall m,m'\in\mathcal{M}. \tag{12k}$$

Outline

Precedence inequalities

- No processing of predecessors after current interval*

$$\sum_{p'=p+1}^{|\mathcal{P}|} d_{j,m}^{p'} \leqslant A_{j,m}^{p}(1 - x_{j,m'}^{p}), \quad \forall (j,m) \prec (j,m'), \forall p \leqslant |\mathcal{P}| - 1. \tag{13}$$



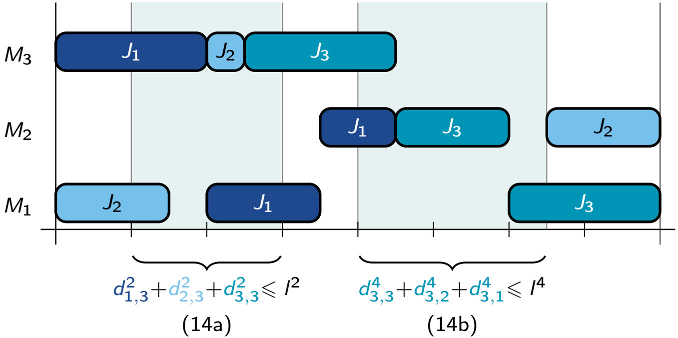
*A similar inequality holds for successors

Non-overlap inequalities

- Same-machine (14a) and same-job (14b) operations non-overlap

$$\sum_{j \in \mathcal{J}} d^{p}_{j,m} \leqslant l^{p}, \qquad \forall m \in \mathcal{M}, \forall p \in \mathcal{P}, \qquad (14a)$$

$$\sum_{m \in \mathcal{M}} d^{p}_{j,m} \leqslant l^{p}, \qquad \forall j \in \mathcal{J}, \forall p \in \mathcal{P}. \qquad (14b)$$

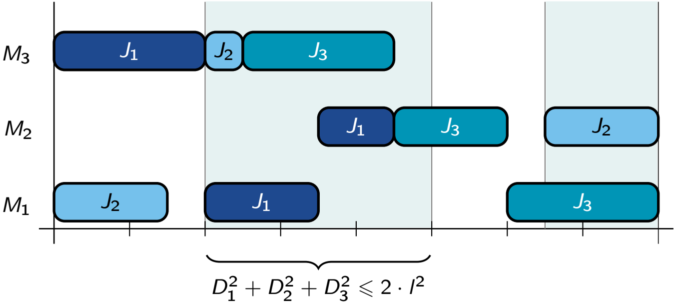


Capacitated non-overlap inequalities

- The processing on a machine of power φ on a period $\equiv$ item of weight φ to pack

$$\sum_{m \in \mathcal{M}} \varphi_m \sum_{j \in \mathcal{J}} d^p_{j,m} \leqslant \overline{\varphi} \cdot l^p, \quad \forall p \in \mathcal{P}. \tag{15}$$

$\varphi = [1, 1, 1], \overline{\varphi} = 2$



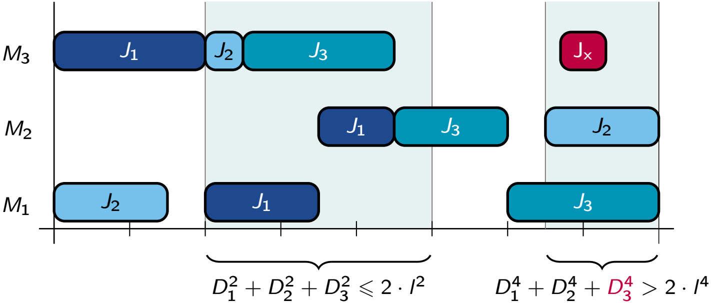
$D^p_m := \sum_{j \in \mathcal{J}} d^p_{j,m}$: total processing on m.

Capacitated non-overlap inequalities

- The processing on a machine of power φ on a period $\equiv$ item of weight φ to pack

$$\sum_{m \in \mathcal{M}} \varphi_m \sum_{j \in \mathcal{J}} d_{j,m}^p \leqslant \overline{\varphi} \cdot l^p, \quad \forall p \in \mathcal{P}. \tag{15}$$

$\varphi = [1, 1, 1], \overline{\varphi} = 2$

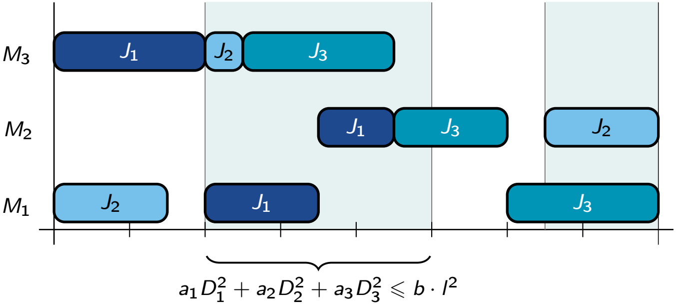


$D_m^p := \sum_{j \in \mathcal{J}} d_{j,m}^p$: total processing on m.

Capacitated non-overlap inequalities

- If $a^\intercal x \leqslant b$ valid for $\text{conv}\{\text{KP}^{\varphi,\overline{\varphi}}\}$ then

$$\sum_{m \in \mathcal{M}} a_m \sum_{j \in \mathcal{J}} d^p_{j,m} \leqslant b \cdot l^p, \quad \forall p \in \mathcal{P}. \tag{15}$$



$D^p_m := \sum_{j \in \mathcal{J}} d^p_{j,m}$: total processing on m.

Other inequalities

- Lifted flow cover inequalities by considering $\overline{E}_m^p := \varphi_m \sum_j d_{j,m}^p / l^p$ and X_m^p.
- If integer input data, then integer processing duration variables*

$$x_{j,m}^p \leqslant d_{j,m}^p, \quad \forall (j,m) \in \mathcal{O}, \forall p \in \mathcal{P}. \tag{16}$$

- Bounds on number of processing intervals **

$$\lceil \frac{q_{j,m}}{\max_p l^p} \rceil \leqslant \sum_{p \in \mathcal{P}} x_{j,m}^p \leqslant \lfloor \frac{q_{j,m}}{\min_p l^p} \rfloor + 2, \quad \forall (j,m) \in \mathcal{O}. \tag{17}$$

$X_m^p :=$ machine status
*better bound propagation with IB cuts
**similar to cont. bin-packing relaxation

Outline

Outline

Instance features and settings

→ Branch-and-Cut using only **minimal covers** and **valid inequalities**.

Instances in Masmoudi et al. [6], based on:

- 2 classic benchmark JSSP instances
  - » ft06 [20] with 6 machines and 6 jobs, $C_{\text{max}} = 56$,
  - » la04 [21] with 5 machines and 10 jobs, $C_{\text{max}} = 590$.
- 3 time horizons $C = \lambda \cdot C_{\text{max}}$ with $\lambda \in \{1.0, 1.1, 1.2\}$,
- 2 Peak power limits $\overline{\varphi} = \alpha \sum_{m \in \mathcal{M}} \varphi_m$ with $\alpha \in \{0.7, 0.9\}$,
- 5 sets of power values φ from $\mathcal{U}[5,10]$,
- On-off peak ToU profile [6]

⇒ 30 small and 30 large instances.

Instance features and settings

$\rightarrow$ Branch-and-Cut using only **minimal covers** and **valid inequalities**.

Instances in Masmoudi et al. [6], based on:

- 2 classic benchmark JSSP instances
  - ft06 [20] with 6 machines and 6 jobs, $C_{\max} = 56$,
  - la04 [21] with 5 machines and 10 jobs, $C_{\max} = 590$.
- 3 time horizons $C = \lambda \cdot C_{\max}$ with $\lambda \in \{1.0, 1.1, 1.2\}$,
- 2 Peak power limits $\overline{\varphi} = \alpha \sum_{m \in \mathcal{M}} \varphi_m$ with $\alpha \in \{0.7, 0.9\}$,
- 5 sets of power values φ from $\mathcal{U}[5,10]$,
- On-off peak ToU profile [6]

$\Rightarrow$ 30 small and 30 large instances.

`Gurobi` on Intel Xeon Gold 6132 (1 thread, 3600s TL) + `Polymake` for $\text{conv}\{\text{KP}^{\varphi,\overline{\varphi}}\}$.

Valid inequalities: root relaxation

TI : Time-Indexed formulation

B&C$_2$: basic and capacitated non-overlap inequalities (14a,14b,15)

		TI			B&C$_2$			relative ($\frac{B\&C_2}{TI}$)	
dim/α/λ	C	T/(%)	$\%^{root}_{bks}$	$\#x^\star/\bar{x}$	T/(%)	$\%^{root}_{bks}$	$\#x^\star/\bar{x}$	#cols	#rows
6x6/0.7/1.0	57	3.3	0.27%	5/5	**1.1**	0.27%	5/5	0.93	0.79
6x6/0.7/1.2	69	12.4	0.06%	5/5	**4.0**	0.06%	5/5	0.85	0.71
6x6/0.9/1.0	55	0.3	0.21%	5/5	**0.1**	0.18%	5/5	0.95	0.81
6x6/0.9/1.2	66	1.0	0.33%	5/5	**1.0**	0.33%	5/5	0.87	0.72
5x10/0.7/1.0	865	-	0.00%	0/0	**89.8**	0.00%	5/5	0.10	0.07
5x10/0.7/1.2	1040	2046.2	0.00%	5/5	**13.2**	0.00%	5/5	0.09	0.07
5x10/0.9/1.0	680	2756.5	0.00%	1/1	**49.2**	0.00%	5/5	0.12	0.09
5x10/0.9/1.2	820	1671.4/(0.32%)	0.00%	4/5	**4.0**	0.00%	5/5	0.10	0.08

Table: Comparison of the variant B&C$_2$ and the TI formulation [6]

- Time to optimality/(gap% at TL): B&C$_2$ **is faster**

Valid inequalities: root relaxation

TI : Time-Indexed formulation

B&C_2 : basic and capacitated non-overlap inequalities (14a,14b,15)

		TI			B&C_2			relative ($\frac{B\&C_2}{TI}$)	
dim/α/λ	C	T/(%)	$\%^{root}_{bks}$	$\#x^\star/\bar{x}$	T/(%)	$\%^{root}_{bks}$	$\#x^\star/\bar{x}$	#cols	#rows
6x6/0.7/1.0	57	3.3	0.27%	5/5	1.1	**0.27%**	5/5	0.93	0.79
6x6/0.7/1.2	69	12.4	0.06%	5/5	4.0	**0.06%**	5/5	0.85	0.71
6x6/0.9/1.0	55	0.3	0.21%	5/5	0.1	**0.18%**	5/5	0.95	0.81
6x6/0.9/1.2	66	1.0	0.33%	5/5	1.0	**0.33%**	5/5	0.87	0.72
5x10/0.7/1.0	865	-	0.00%	0/0	89.8	**0.00%**	5/5	0.10	0.07
5x10/0.7/1.2	1040	2046.2	0.00%	5/5	13.2	**0.00%**	5/5	0.09	0.07
5x10/0.9/1.0	680	2756.5	0.00%	1/1	49.2	**0.00%**	5/5	0.12	0.09
5x10/0.9/1.2	820	1671.4/(0.32%)	0.00%	4/5	4.0	**0.00%**	5/5	0.10	0.08

Table: Comparison of the variant B&C_2 and the TI formulation [6]

- Time to optimality/(gap% at TL): B&C_2 is faster
- Root relaxation strength: B&C_2 **has strong root relaxations**

Valid inequalities: root relaxation

TI : Time-Indexed formulation

B&C_2 : basic and capacitated non-overlap inequalities (14a,14b,15)

		TI			B&C_2			relative ($\frac{B\&C_2}{TI}$)	
dim/α/λ	C	T/(%)	$\%^{root}_{bks}$	$\#x^\star/\bar{x}$	T/(%)	$\%^{root}_{bks}$	$\#x^\star/\bar{x}$	#cols	#rows
6x6/0.7/1.0	57	3.3	0.27%	5/5	1.1	0.27%	**5/5**	0.93	0.79
6x6/0.7/1.2	69	12.4	0.06%	5/5	4.0	0.06%	**5/5**	0.85	0.71
6x6/0.9/1.0	55	0.3	0.21%	5/5	0.1	0.18%	**5/5**	0.95	0.81
6x6/0.9/1.2	66	1.0	0.33%	5/5	1.0	0.33%	**5/5**	0.87	0.72
5x10/0.7/1.0	865	-	0.00%	0/0	89.8	0.00%	**5/5**	0.10	0.07
5x10/0.7/1.2	1040	2046.2	0.00%	5/5	13.2	0.00%	**5/5**	0.09	0.07
5x10/0.9/1.0	680	2756.5	0.00%	1/1	49.2	0.00%	**5/5**	0.12	0.09
5x10/0.9/1.2	820	1671.4/(0.32%)	0.00%	4/5	4.0	0.00%	**5/5**	0.10	0.08

Table: Comparison of the variant B&C_2 and the TI formulation [6]

- Time to optimality/(gap% at TL): B&C_2 is faster
- Root relaxation strength: : B&C_2 has strong root relaxations
- Model size: B&C_2 **scales better**

Valid inequalities: root relaxation

TI : Time-Indexed formulation

B&C$_2$: basic and capacitated non-overlap inequalities (14a,14b,15)

		TI			B&C$_2$			relative ($\frac{B\&C_2}{TI}$)	
dim/α/λ	C	T/(%)	$\%^{root}_{bks}$	$\#x^\star/\bar{x}$	T/(%)	$\%^{root}_{bks}$	$\#x^\star/\bar{x}$	#cols	#rows
6x6/0.7/1.0	57	3.3	0.27%	5/5	1.1	0.27%	5/5	**0.93**	**0.79**
6x6/0.7/1.2	69	12.4	0.06%	5/5	4.0	0.06%	5/5	**0.85**	**0.71**
6x6/0.9/1.0	55	0.3	0.21%	5/5	0.1	0.18%	5/5	**0.95**	**0.81**
6x6/0.9/1.2	66	1.0	0.33%	5/5	1.0	0.33%	5/5	**0.87**	**0.72**
5x10/0.7/1.0	865	-	0.00%	0/0	89.8	0.00%	5/5	**0.10**	**0.07**
5x10/0.7/1.2	1040	2046.2	0.00%	5/5	13.2	0.00%	5/5	**0.09**	**0.07**
5x10/0.9/1.0	680	2756.5	0.00%	1/1	49.2	0.00%	5/5	**0.12**	**0.09**
5x10/0.9/1.2	820	1671.4/(0.32%)	0.00%	4/5	4.0	0.00%	5/5	**0.10**	**0.08**

Table: Comparison of the variant B&C$_2$ and the TI formulation [6]

- Time to optimality/(gap% at TL): B&C$_2$ is faster
- Root relaxation strength: : B&C$_2$ has strong root relaxations
- Model size: B&C$_2$ scales better
- Number of optimal/feasible solutions: : B&C$_2$ **solves all to optimality**

Instance features and settings

$\rightarrow$ Branch-and-Cut using only **minimal covers** and **valid inequalities**.

Extend previous instances with:

- JSSP instances with ∼50 operations generated as in Lawrence [21],
- 3 time horizons $C = \lambda \cdot C_{\max}$ with $\lambda \in \{1.05, 1.1, 1.2\}$,
- 2 Peak power limits $\overline{\varphi} = \alpha \sum_{m \in \mathcal{M}} \varphi_m$ with $\alpha \in \{0.6, 0.7\}$,
- 4 sets of power values,
- On-off peak ToU profile [6].

$\Rightarrow$ 72 instances.

`Gurobi` on Intel Xeon Gold 6132 (1 thread, 3600s TL) + `Polymake` for conv$\{\mathrm{KP}^{\varphi,\overline{\varphi}}\}$.

Valid inequalities: tree exploration

B&C$_2$: basic and capacitated non-overlap inequalities

B&C$_{all}$: all valid inequalities

	B&C$_2$			B&C$_{all}$		
dim/λ	T	#nd	#x*	T	#nd	#x*
5x10/1.05	82.9	8510	9	**125.5**	8649	9
5x10/1.1	67.0	5888	9	**50.2**	2331	10
5x10/1.2	120.4	9385	9	**86.2**	3145	9
6x8/1.05	66.7	8627	8	**33.6**	2301	8
6x8/1.1	51.0	5152	8	**55.1**	2760	8
6x8/1.2	34.8	3584	8	**18.1**	1080	8
7x7/1.05	385.7	51k	6	**330.8**	35k	6
7x7/1.1	119.4	19k	7	**155.8**	18k	7
7x7/1.2	60.1	8984	7	**54.1**	4297	7
all (78)	86.16	9613	71	**74.81**	4783	72

Table: Comparison with variant including all valid inequalities.

- Time to optimality: B&C$_{all}$ **is slightly faster**

Valid inequalities: tree exploration

B&C_2 : basic and capacitated non-overlap inequalities   B&C_{all} : all valid inequalities

	B&C_2			B&C_{all}		
dim/λ	T	#nd	#$x^\star$	T	#nd	#$x^\star$
5x10/1.05	82.9	8510	9	125.5	**8649**	9
5x10/1.1	67.0	5888	9	50.2	**2331**	10
5x10/1.2	120.4	9385	9	86.2	**3145**	9
6x8/1.05	66.7	8627	8	33.6	**2301**	8
6x8/1.1	51.0	5152	8	55.1	**2760**	8
6x8/1.2	34.8	3584	8	18.1	**1080**	8
7x7/1.05	385.7	51k	6	330.8	**35k**	6
7x7/1.1	119.4	19k	7	155.8	**18k**	7
7x7/1.2	60.1	8984	7	54.1	**4297**	7
all (78)	86.16	9613	71	74.81	**4783**	72

Table: Comparison with variant including all valid inequalities.

- Time to optimality: B&C_{all} is slightly faster
- Number of explored nodes: B&C_{all} **explores half the number of nodes**

Valid inequalities: tree exploration

B&C$_2$: basic and capacitated non-overlap inequalities

B&C$_{all}$: all valid inequalities

	B&C$_2$			B&C$_{all}$		
dim/λ	T	#nd	#x*	T	#nd	#x*
5x10/1.05	82.9	8510	9	125.5	8649	**9**
5x10/1.1	67.0	5888	9	50.2	2331	**10**
5x10/1.2	120.4	9385	9	86.2	3145	**9**
6x8/1.05	66.7	8627	8	33.6	2301	**8**
6x8/1.1	51.0	5152	8	55.1	2760	**8**
6x8/1.2	34.8	3584	8	18.1	1080	**8**
7x7/1.05	385.7	51k	6	330.8	35k	**6**
7x7/1.1	119.4	19k	7	155.8	18k	**7**
7x7/1.2	60.1	8984	7	54.1	4297	**7**
all (78)	86.16	9613	71	74.81	4783	**72**

Table: Comparison with variant including all valid inequalities.

- Time to optimality: B&C$_{all}$ is slightly faster
- Number of explored nodes: B&C$_{all}$ explores half the number of nodes
- Number of optimal solutions: B&C$_{all}$ **solves one more instance to optimality**

Outline

Instance features and settings

→ $\texttt{B\&C}_{\texttt{all}}$ with different **clique-forbidding** cuts.

New instances with:

- JSSP instances with ~60 operations generated as in Lawrence [21],
- 3 time horizons $C = \lambda \cdot C_{\max}$ with $\lambda \in \{1.05, 1.1, 1.2\}$,
- 2 Peak power limits $\overline{\varphi} = \alpha \sum_{m \in \mathcal{M}} \varphi_m$ with $\alpha \in \{0.6, 0.7\}$,
- 4 sets of power values ($KP^{\varphi,\overline{\varphi}}$)
  - `sL`: small and large coefficients $a_{\mathcal{I}} \ll a_{\mathcal{J}}$,
  - `WSI`: weakly super increasing coefficients $a_i | a_{i+1}$,
  - `KP3`: three distinct coefficients $|\{a\}| = 3$,
  - `1-AS`: arithmetic sequence coefficients $a_{i+1} - a_i = 1$.
- On-off peak ToU profile [6]

⇒ 72 instances.

Clique-forbidding cuts

B&C$_{\texttt{mcv}}$: only minimal cover inequalities

B&C$_{\texttt{mcv+fi}}$: minimal cover and FI inequalities

		B&C$_{\texttt{mcv}}$			B&C$_{\texttt{mcv+fi}}$		
KP type	FI char.	T	$\#x^\star$	#cuts	T	$\#x^\star$	#cuts
sL	mcv+lci	242.9	14	577	244.43	14	652
WSI	mcv	415.1	16	190	**393.5**	**16**	207
KP3	ext	**288.1**	**12**	817	314.9	12	949
1-AS	lci	377.1	15	879	**258.25**	**16**	838
all [72]	-	323.7	57	529	**297.6**	**58**	571

Table: Comparison with variant including all clique-forbidding inequalities.

- **FI inequalities characterizing conv$\{KP^{\varphi,\overline{\varphi}}\}$**

Clique-forbidding cuts

$\texttt{B\&C}_{\texttt{mcv}}$: only minimal cover inequalities

$\texttt{B\&C}_{\texttt{mcv+fi}}$: minimal cover and FI inequalities

		$\texttt{B\&C}_{\texttt{mcv}}$			$\texttt{B\&C}_{\texttt{mcv+fi}}$		
KP type	FI char.	T	#x*	#cuts	T	#x*	#cuts
sL	mcv+lci	242.9	14	577	244.43	14	652
WSI	mcv	415.1	16	190	**393.5**	**16**	207
KP3	ext	**288.1**	**12**	817	314.9	12	949
1-AS	lci	377.1	15	879	**258.25**	**16**	838
all [72]	-	323.7	57	529	**297.6**	**58**	571

Table: Comparison with variant including all clique-forbidding inequalities.

- FI inequalities characterizing $\text{conv}\{KP^{\varphi,\overline{\varphi}}\}$
- Time to optimality: $\texttt{B\&C}_{\texttt{mcv+fi}}$ **is faster on average**

Clique-forbidding cuts

$\texttt{B\&C}_{\texttt{mcv}}$: only minimal cover inequalities

$\texttt{B\&C}_{\texttt{mcv+fi}}$: minimal cover and FI inequalities

		$\texttt{B\&C}_{\texttt{mcv}}$			$\texttt{B\&C}_{\texttt{mcv+fi}}$		
KP type	FI char.	T	#x*	#cuts	T	#x*	#cuts
sL	mcv+lci	242.9	14	577	244.43	14	652
WSI	mcv	415.1	16	190	**393.5**	**16**	207
KP3	ext	**288.1**	**12**	817	314.9	12	949
1-AS	lci	377.1	15	879	**258.25**	**16**	838
all [72]	-	323.7	57	529	**297.6**	**58**	571

Table: Comparison with variant including all clique-forbidding inequalities.

- FI inequalities characterizing $\text{conv}\{KP^{\varphi,\overline{\varphi}}\}$
- Time to optimality: $\texttt{B\&C}_{\texttt{mcv+fi}}$ is faster on average
- Number of optimal solutions: $\texttt{B\&C}_{\texttt{mcv+fi}}$ **solves one more instance to optimality**

Outline

1. Introduction

2. Branch-and-Cut approach

3. Computational results

4. Conclusion and perspectives

Conclusions

- A new model the $Jm|\mathrm{P}_{\max}|\mathrm{TEC}$, indexed on the ToU profile periods.
- Peak power constraint separation by forbidding cliques in a B&C algorithm.
  - » using minimal cover inequalities of a knapsack set.
  - » using (any) inequalities describing its polytope.
- Different families of valid inequalities were explored.
- Compared to the SoA Time-Indexed formulation:
  - ✓ more compact,
  - ✓ as strong linear relaxations,
  - ✓ solves all previously unsolved instances and most of the newly proposed ones.

Research perspectives and future work

→ Find α^e and β s.t. other cover inequalities are model-defining ($\Longleftrightarrow$).

Research perspectives and future work

→ Find α^e and β s.t. other cover inequalities are model-defining ($\Longleftrightarrow$).

→ Characterizing hard instances for Period-Indexed (PI) formulations.

→ Studying PI formulations relative to the ToU profile.

→ Characterizing strong valid inequalities for PI formulations.

Research perspectives and future work

→ Find α^e and β s.t. other cover inequalities are model-defining ($\Longleftrightarrow$).

→ Characterizing hard instances for Period-Indexed (PI) formulations.

→ Studying PI formulations relative to the ToU profile.

→ Characterizing strong valid inequalities for PI formulations.

→ Generalizing to other shop-scheduling problems under ToU pricing.

→ Solving the $Jm|\mathrm{P}_{\max}|\mathrm{TEC}$ for any general cost function.

References I

[1] Michael R Garey, David S Johnson, and Ravi Sethi. The complexity of flowshop and jobshop scheduling. *Mathematics of operations research*, 1(2):117–129, 1976.

[2] Clean Energy Alliance. Seasonal time-of-use pricing schemes, 2024. URL `https://web.archive.org/web/20241202083629/https://thecleanenergyalliance.org/time-of-use-pricing/`.

[3] R. Graham, E. Lawler, J. Lenstra, and A. Kan. Optimization and approximation in deterministic sequencing and scheduling: A survey. *Annals of Discrete Mathematics*, 5:287–326, 1977.

[4] Marouane Felloussi, Xavier Delorme, and Paolo Gianessi. Minimizing energy cost in a job-shop scheduling problem under ToU pricing: a new method based on a period-indexed MILP. In *Proceedings of the 14th International Conference on Operations Research and Enterprise Systems - ICORES*. INSTICC, SciTePress (to appear), 2025.

[5] Andreas Bley and Andreas Linß. Propagation and branching strategies for job shop scheduling minimizing the weighted energy consumption. In *International Conference on Operations Research*, pages 573–580. Springer, 2022.

References II

[6] Oussama Masmoudi, Xavier Delorme, and Paolo Gianessi. Job-shop scheduling problem with energy consideration. *International Journal of Production Economics*, 216:12–22, 10 2019. ISSN 09255273. doi: 10.1016/j.ijpe.2019.03.021.

[7] Myoung Ju Park and Andy Ham. Energy-aware flexible job shop scheduling under time-of-use pricing. *International Journal of Production Economics*, 248, 6 2022. ISSN 09255273. doi: 10.1016/j.ijpe.2022.108507.

[8] Enda Jiang and Ling Wang. Multi-objective optimization based on decomposition for flexible job shop scheduling under time-of-use electricity prices. *Knowledge-Based Systems*, 204, 9 2020. ISSN 09507051. doi: 10.1016/j.knosys.2020.106177.

[9] Minh Hung Ho, Faicel Hnaien, and Frederic Dugardin. Exact method to optimize the total electricity cost in two-machine permutation flow shop scheduling problem under time-of-use tariff. *Computers and Operations Research*, 144, 8 2022. ISSN 03050548. doi: 10.1016/j.cor.2022.105788.

[10] Mauro Gaggero, Massimo Paolucci, and Roberto Ronco. Exact and heuristic solution approaches for energy-efficient identical parallel machine scheduling with time-of-use costs. *European Journal of Operational Research*, 311:845–866, 12 2023. ISSN 03772217. doi: 10.1016/j.ejor.2023.05.040.

References III

[11] Ada Che, Shibohua Zhang, and Xueqi Wu. Energy-conscious unrelated parallel machine scheduling under time-of-use electricity tariffs. *Journal of Cleaner Production*, 156:688–697, 7 2017. ISSN 09596526. doi: 10.1016/j.jclepro.2017.04.018.

[12] Jian Ya Ding, Shiji Song, Rui Zhang, Raymond Chiong, and Cheng Wu. Parallel machine scheduling under time-of-use electricity prices: New models and optimization approaches. *IEEE Transactions on Automation Science and Engineering*, 13:1138–1154, 4 2016. ISSN 15455955. doi: 10.1109/TASE.2015.2495328.

[13] Zheng Tian and Li Zheng. Single machine parallel-batch scheduling under time-of-use electricity prices: New formulations and optimisation approaches. *European Journal of Operational Research*, 312:512–524, 1 2024. ISSN 03772217. doi: 10.1016/j.ejor.2023.07.012.

[14] Junheng Cheng, Feng Chu, Chengbin Chu, and Weili Xia. Bi-objective optimization of single-machine batch scheduling under time-of-use electricity prices. *RAIRO - Operations Research*, 50:715–732, 10 2016. ISSN 28047303. doi: 10.1051/ro/2015063.

[15] Baigang Du, Tian Tan, Jun Guo, Yibing Li, and Shunsheng Guo. Energy-cost-aware resource-constrained project scheduling for complex product system with activity splitting and recombining. *Expert Systems with Applications*, 173:114754, 2021.

References IV

[16] Margaux Nattaf, Markó Horváth, Tamás Kis, Christian Artigues, and Pierre Lopez. Polyhedral results and valid inequalities for the continuous energy-constrained scheduling problem. *Discrete Applied Mathematics*, 258:188–203, 2019.

[17] Paul Turán. Eine extremalaufgabe aus der graphentheorie. *Mat. Fiz. Lapok*, 48(436-452):61 (in Hungarian), 1941.

[18] Harvey Abbott and Meir Katchalski. A Turán type problem for interval graphs. *Discrete Mathematics*, 25(1):85–88, 1979. doi: 10.1016/0012-365X(79)90155-9.

[19] David Applegate and William Cook. A computational study of the job-shop scheduling problem. *ORSA Journal on computing*, 3(2):149–156, 1991. doi: 10.1287/ijoc.3.2.149.

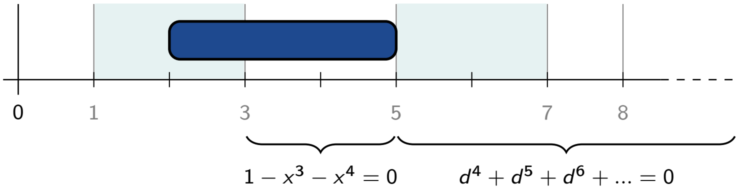
[20] H Fisher and GL Thompson. Probabilistic learning combinations of local job-shop scheduling rules. *Prentice Hall, Englewood Cliffs, New Jersey*, pages 225–251, 1963.

[21] Stephen Lawrence. Resouce constrained project scheduling: An experimental investigation of heuristic scheduling techniques (supplement). *Graduate School of Industrial Administration, Carnegie-Mellon University*, 1984.

Non-preemption inequalities (1)

- No processing on the intervals after operation completion*

$$\sum_{p'=p+1}^{|\mathcal{P}|} d_{j,m}^{p'} \leqslant B_{j,m}^{p}(1 - x_{j,m}^{p} + x_{j,m}^{p+1}), \quad \forall (j,m) \in \mathcal{O}, \forall p \leqslant |\mathcal{P}| - 1. \tag{18}$$

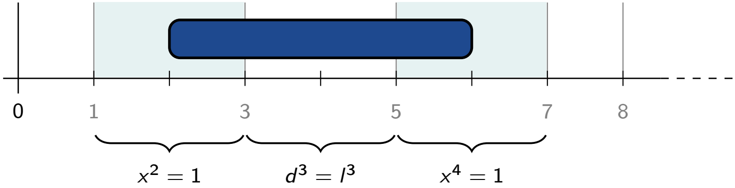


*A similar inequality holds for operation start

Non-preemption inequalities (2)

- No operation interrupts processing

$$d_{j,m}^{p} \geqslant l^{p}(x_{j,m}^{p_1} + x_{j,m}^{p_2} - 1), \quad \forall (j,m) \in \mathcal{O}, \forall p_1 < p < p_2 \in \mathcal{P}. \tag{19}$$



Problem and instances

JSSP under ToU pricing:

- No peak power limit constraint,
- Period-Indexed formulation with basic non-overlap inequalities (PI_2).

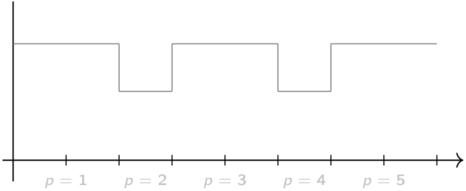
Problem and instances

JSSP under ToU pricing:

- No peak power limit constraint,
- Period-Indexed formulation with basic non-overlap inequalities (PI_2).

Subset of instances taken from [6]:

- la04 with 5 machines and 10 jobs, $C_{max} = 590$.
- ToU profiles
  - » **on-off peak (ratio 2:1),**
  - » on-off peak (ratio 1:2),
  - » on-off peak (ratio 1:1),
  - » mid-off-on peak.



Gurobi on an Intel Xeon Gold 6132 (1 thread, 3600s TL).

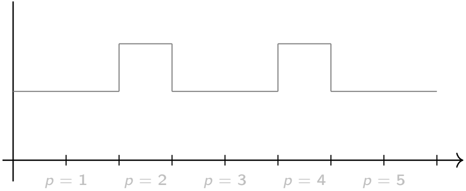
Problem and instances

JSSP under ToU pricing:

- No peak power limit constraint,
- Period-Indexed formulation with basic non-overlap inequalities (PI_2).

Subset of instances taken from [6]:

- la04 with 5 machines and 10 jobs, $C_{\max} = 590$.
- ToU profiles
  - » on-off peak (ratio 2:1),
  - » **on-off peak (ratio 1:2),**
  - » on-off peak (ratio 1:1),
  - » mid-off-on peak.



Gurobi on an Intel Xeon Gold 6132 (1 thread, 3600s TL).

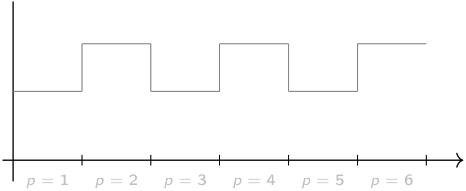
Problem and instances

JSSP under ToU pricing:

- No peak power limit constraint,
- Period-Indexed formulation with basic non-overlap inequalities (PI_2).

Subset of instances taken from [6]:

- la04 with 5 machines and 10 jobs, $C_{max} = 590$.
- ToU profiles
  - on-off peak (ratio 2:1),
  - on-off peak (ratio 1:2),
  - **on-off peak (ratio 1:1),**
  - mid-off-on peak.



Gurobi on an Intel Xeon Gold 6132 (1 thread, 3600s TL).

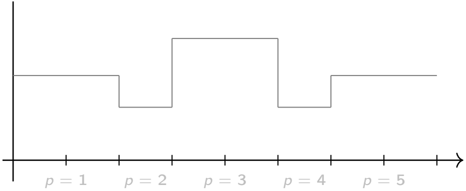
Problem and instances

JSSP under ToU pricing:

- No peak power limit constraint,
- Period-Indexed formulation with basic non-overlap inequalities (PI_2).

Subset of instances taken from [6]:

- la04 with 5 machines and 10 jobs, $C_{max} = 590$.
- ToU profiles
  - » on-off peak (ratio 2:1),
  - » on-off peak (ratio 1:2),
  - » on-off peak (ratio 1:1),
  - » **mid-off-on peak.**



Gurobi on an Intel Xeon Gold 6132 (1 thread, 3600s TL).

ToU profile impact

ToU profile	TI			PI_2		
	T/(%)	$\%_{bks}^{root}$	#nd	T/(%)	$\%_{bks}^{root}$	#nd
on-off (2:1)	2564.31	0.0%	28	**0.14**	0.0%	1
on-off (1:2)	3103.69	0.4%	925	**67.89**	1.0%	35K
on-off (1:1)	(0.11%)*	0.1%	407	**419.35**	0.1%	168K
mid-on-off	2406.81	0.5%	631	**84.92**	1.6%	12K

Table: Comparison of TI and PI_2 on different ToU profiles.

- Time to optimality/(gap% at TL): **sensitivity to profile but PI_2 better**

*#opt. = 0

ToU profile impact

ToU profile	TI T/(%)	TI $\%_{bks}^{root}$	TI #nd	PI_2 T/(%)	PI_2 $\%_{bks}^{root}$	PI_2 #nd
on-off (2:1)	2564.31	**0.0%**	28	0.14	0.0%	1
on-off (1:2)	3103.69	**0.4%**	925	67.89	1.0%	35K
on-off (1:1)	(0.11%)*	**0.1%**	407	419.35	0.1%	168K
mid-on-off	2406.81	**0.5%**	631	84.92	1.6%	12K

Table: Comparison of TI and PI_2 on different ToU profiles.

- Time to optimality/(gap% at TL): sensitivity to profile but PI_2 better
- Root relaxation strength: **minor impact on both**

*#opt. = 0

ToU profile impact

ToU profile	TI T/(%)	TI $\%^{root}_{bks}$	TI #nd	PI_2 T/(%)	PI_2 $\%^{root}_{bks}$	PI_2 #nd
`on-off (2:1)`	2564.31	0.0%	**28**	0.14	0.0%	**1**
`on-off (1:2)`	3103.69	0.4%	**925**	67.89	1.0%	**35K**
`on-off (1:1)`	(0.11%)*	0.1%	**407**	419.35	0.1%	**168K**
`mid-on-off`	2406.81	0.5%	**631**	84.92	1.6%	**12K**

Table: Comparison of TI and PI_2 on different ToU profiles.

- Time to optimality/(gap% at TL): sensitivity to profile but PI_2 better
- Root relaxation strength: minor impact on both
- Number of explored nodes: **more time spent per node**

*#opt. = 0

ToU profile impact

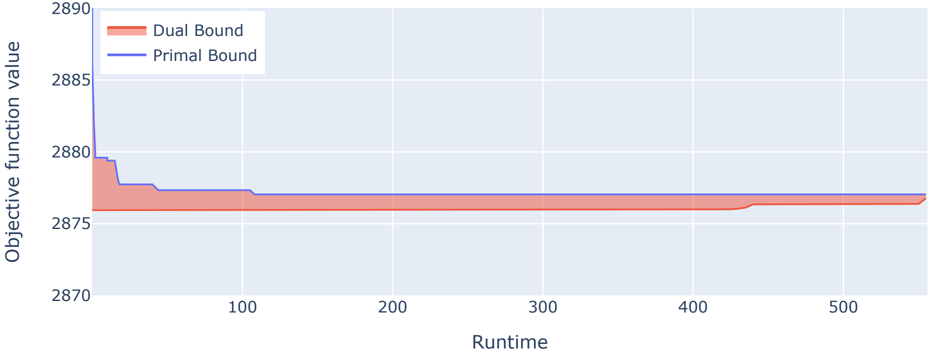


Figure: UB and LB progress on an example instance with on-off (1:1) profile