The Energy-Aware Job-Shop Scheduling Problem under Time-of-Use Pricing

A Period-Indexed model

Marouane Felloussi, Xavier Delorme, Paolo Gianessi

Mines Saint-Etienne, LIMOS UMR 6158 CNRS,
Saint-Etienne, France

February 24, 2025

ICORES 2025, Porto, Portugal

Outline

Outline

Energy-Aware Job-Shop Scheduling

The Job-Shop Scheduling Problem:

- Schedule jobs with ordered operations on machines to optimize some criterion.
- Relevant problem in OR, strongly NP-hard [1].
- Modeling Approaches:
  - » Time-Indexed formulations ⇒ strong dual bounds, large MILPs.
  - » Disjunctive formulations ⇒ more compact MILPs, weak relaxations.

Energy-Aware Job-Shop Scheduling

Energy consideration:

- Production is energy-intensive, electrification of processes (etc.)
- ToU pricing:
  - » Non-regular criterion.
  - » Period-Indexed formulation

Figure: Seasonal Time-of-Use pricing example [2]

Problem Definition

- **Machines** $m \in \mathcal{M}$ with power consumption φ_m.
- **Jobs** $j \in \mathcal{J}$ to execute over a time horizon C:
  - » Processed over an ordered subset of machines.
  - » Having constant processing times $q_{j,m} \Rightarrow$ energy consumption $\varphi_m \cdot q_{j,m}$.
  - » Direct precedence constraints.
- **Operations** $o \in \mathcal{O} := \mathcal{J} \times \mathcal{M}$, i.e. processing of a job on a machine.
- **ToU Periods** $p \in \mathcal{P}$:
  - » Duration l^p and electricity price c_p per unit.
  - » $[t^p, t^{p+1}]$, with $t^{p+1} - t^p = l_p$, $t^1 = 0$ and $t^{|\mathcal{P}|} = C$.

Problem Definition

A **feasible solution** consists of a **schedule** where each job $j \in \mathcal{J}$ processes over machines $m \in \mathcal{M}$, during one or more ToU periods in $\mathcal{P}$, such that:

- same-machine operations do not process simultaneously **(non-overlap)**,
- operations may not interrupt processing **(non-preemption)**,
- operation sequencing respects a predefined order **(precedence)**.

Objective

The goal is to find a **feasible** solution **minimizing** the Total Energy Cost **(TEC)**.

Instance and Solution

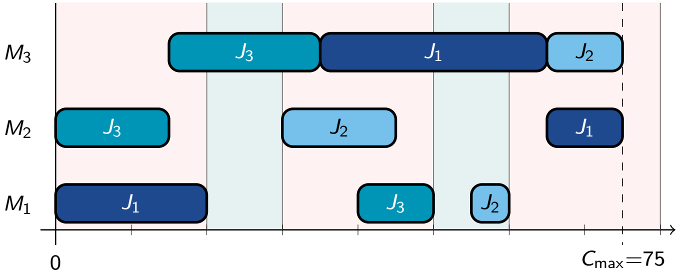
Machine m	1	2	3
Power φ_m	5	6	8

Job j	1	2	3
Sequence M_j	$\{1, 3, 2\}$	$\{2, 1, 3\}$	$\{2, 3, 1\}$

Machine m	1	2	3
$q_{1,m}$	20	10	30
$q_{2,m}$	5	15	10
$q_{3,m}$	10	15	20

ToU period p	on-peak	off-peak
Tariff c^p	0.159	0.13
Duration l^p	20	10

Makespan minimization (TEC $=$ 208.2)



Instance and Solution

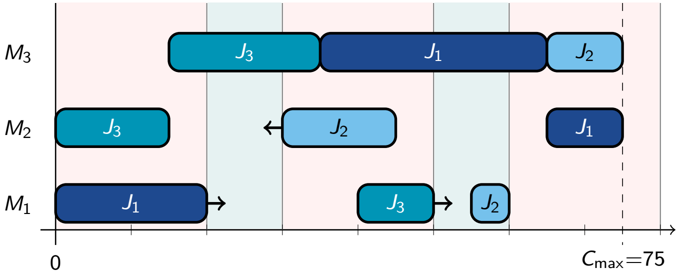
Machine m	1	2	3
Power φ_m	5	6	8

Job j	1	2	3
Sequence M_j	$\{1,3,2\}$	$\{2,1,3\}$	$\{2,3,1\}$

Machine m	1	2	3
$q_{1,m}$	20	10	30
$q_{2,m}$	5	15	10
$q_{3,m}$	10	15	20

ToU period p	on-peak	off-peak
Tariff c^p	0.159	0.13
Duration l^p	20	10

Makespan minimization (TEC = 208.2)



Instance and Solution

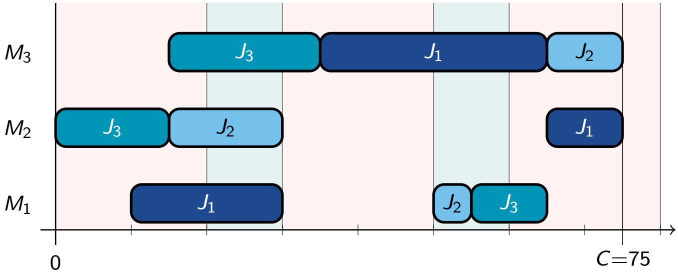
Machine m	1	2	3
Power φ_m	5	6	8

Job j	1	2	3
Sequence M_j	$\{1,3,2\}$	$\{2,1,3\}$	$\{2,3,1\}$

Machine m	1	2	3
$q_{1,m}$	20	10	30
$q_{2,m}$	5	15	10
$q_{3,m}$	10	15	20

ToU period p	on-peak	off-peak
Tariff c^p	0.159	0.13
Duration l^p	20	10

Energy cost minimization s.t makespan constraint $C_{\max}$ (TEC $=$ 199.9)



Short literature review

Problem class	Article	Problem*	MILP Formulation
job-shop scheduling	[4]	$Jm \vert$ on/off, $r_j, d_j \vert$ TEC	TI
	[5]	$Jm \vert \mathrm{P_{max}} \vert$ TEC	D,TI
flexible job-shop scheduling	[6]	$FJm \vert$ on/off $\vert C_{max}$, TEC	TI
	[7]	$FJm \vert\vert C_{max}$, TEC	TI
flow-shop scheduling	[8]	$F2 \vert prmu$, on/off $\vert$ TEC	PI+TI
parallel machine scheduling	[9]	$Pm \vert\vert C_{max}$, TEC	TI
	[10]	$Rm \vert\vert$ TEC	PI
	[11]	$Rm \vert\vert$ TEC	PI
single machine scheduling	[12]	$1 \vert batch \vert$ TEC	PI,TI
	[13]	$1 \vert batch \vert$ TEC	PI

Table: Some works on energy-aware shop and machine scheduling.

*Graham's 3-field notation [3]

Short literature review

Problem class	Article	Problem*	MILP Formulation
job-shop scheduling	[4]	$Jm\|$ on/off, $r_j, d_j\|$TEC	TI
	[5]	$Jm\|\mathrm{P}_{\max}\|$TEC	D,TI
flexible job-shop scheduling	[6]	$FJm\|$ on/off $\|C_{\max}$,TEC	TI
	[7]	$FJm\|\|C_{\max}$,TEC	TI
flow-shop scheduling	[8]	$F2\|prmu$, on/off $\|$TEC	PI+TI
parallel machine scheduling	[9]	$Pm\|\|C_{\max}$,TEC	TI
	[10]	$Rm\|\|$TEC	PI
	[11]	$Rm\|\|$TEC	PI
single machine scheduling	[12]	$1\|batch\|$TEC	PI,TI
	[13]	$1\|batch\|$TEC	PI

Table: Some works on energy-aware shop and machine scheduling.

*Graham's 3-field notation [3]

Short literature review

Problem class	Article	Problem*	MILP Formulation
job-shop scheduling	*this work*	$Jm\|\|\text{TEC}$	PI ⟵
	[4]	$Jm\|\text{on/off}, r_j, d_j\|\text{TEC}$	TI
	[5]	$Jm\|\text{P}_{\text{max}}\|\text{TEC}$	D,TI
flexible job-shop scheduling	[6]	$FJm\|\text{on/off}\|C_{\text{max}},\text{TEC}$	TI
	[7]	$FJm\|\|C_{\text{max}},\text{TEC}$	TI
flow-shop scheduling	[8]	$F2\|prmu, \text{on/off}\|\text{TEC}$	PI+TI
parallel machine scheduling	[9]	$Pm\|\|C_{\text{max}},\text{TEC}$	TI
	[10]	$Rm\|\|\text{TEC}$	PI
	[11]	$Rm\|\|\text{TEC}$	PI
single machine scheduling	[12]	$1\|batch\|\text{TEC}$	PI,TI
	[13]	$1\|batch\|\text{TEC}$	PI

Table: Some works on energy-aware shop and machine scheduling.

*Graham's 3-field notation [3]

Main contributions

We introduce:

- a **new period-indexed MILP** for the $Jm||\text{TEC}$,
- **valid inequalities** to improve linear relaxations and B&B tree exploration.

We show the results of computational experiments aimed to:

- compare the **period-indexed formulation** against the state of the art,
- assess the effectiveness of the **valid inequalities**,
- show the impact of the **Time-of-Use** profile.

Outline

Variables

Processing status

$$x_{j,m}^{p} = \begin{cases} 1, & \text{if operation } (j,m) \text{ is processed during period } p \\ 0, & \text{otherwise} \end{cases} \qquad \forall j \in \mathcal{J}, m \in \mathcal{M}, p \in \mathcal{P}$$

Processing duration

$d_{j,m}^{p} \in \mathbb{R}^{+}$: time spent processing operation (j,m) on period p. $\quad \forall j \in \mathcal{J}, m \in \mathcal{M}, p \in \mathcal{P}$

Variables

Processing status

$$x^p_{j,m} = \begin{cases} 1, & \text{if operation } (j, m) \text{ is processed during period } p \\ 0, & \text{otherwise} \end{cases} \qquad \forall j \in \mathcal{J}, m \in \mathcal{M}, p \in \mathcal{P}$$

Processing duration

$d^p_{j,m} \in \mathbb{R}^+$: time spent processing operation (j, m) on period p. $\quad \forall j \in \mathcal{J}, m \in \mathcal{M}, p \in \mathcal{P}$

Starting/completion date

$s_{j,m}, c_{j,m} \in \mathbb{R}^+$: starting and completion dates of operation (j, m) $\quad \forall j \in \mathcal{J}, m \in \mathcal{M}$

Machine disjunction

$$u_{j,j',m} = \begin{cases} 1, & \text{processing of operation } (j, m) \text{ ends before start of } (j', m) \\ 0, & \text{otherwise} \end{cases} \qquad \forall j < j' \in \mathcal{J}, m \in \mathcal{M}$$

Objective function

Schedule total cost

The total operational cost of a schedule is minimized:

$$\min \sum_{p \in \mathcal{P}} c^p \sum_{m \in M} \varphi_m \sum_{j \in \mathcal{J}} d^p_{j,m} \tag{1}$$

φ_m: power of machine m
c^p: cost of period p

Core constraints

Total operation processing

$$\sum_{p \in \mathcal{P}} d^{p}_{j,m} = q_{j,m}, \quad \forall (j,m) \in \mathcal{O}. \tag{2}$$

$q_{j,m}$: duration of (j, m)

Core constraints

Total operation processing

$$\sum_{p \in \mathcal{P}} d^{p}_{j,m} = q_{j,m}, \quad \forall (j,m) \in \mathcal{O}. \tag{2}$$

$q_{j,m}$: duration of (j, m)

Machine disjunction

$$c_{j,m} - s_{j',m} \leqslant \alpha_{j,j',m} \cdot (1 - u_{j,j',m}), \qquad \forall m \in \mathcal{M}, \forall j, j' \in \mathcal{J} : j < j', \tag{3a}$$

$$c_{j',m} - s_{j,m} \leqslant \beta_{j,j',m} \cdot u_{j,j',m}, \qquad \forall m \in \mathcal{M}, \forall j, j' \in \mathcal{J} : j < j'. \tag{3b}$$

$\alpha_{j,j',m}$ and $\beta_{j,j',m}$: constants

Core constraints

Total operation processing

$$\sum_{p \in \mathcal{P}} d^p_{j,m} = q_{j,m}, \quad \forall (j,m) \in \mathcal{O}. \tag{2}$$

$q_{j,m}$: duration of (j, m)

Machine disjunction

$$c_{j,m} - s_{j',m} \leqslant \alpha_{j,j',m} \cdot (1 - u_{j,j',m}), \qquad \forall m \in \mathcal{M}, \forall j, j' \in \mathcal{J} : j < j', \tag{3a}$$

$$c_{j',m} - s_{j,m} \leqslant \beta_{j,j',m} \cdot u_{j,j',m}, \qquad \forall m \in \mathcal{M}, \forall j, j' \in \mathcal{J} : j < j'. \tag{3b}$$

$\alpha_{j,j',m}$ and $\beta_{j,j',m}$: constants

Precedence

$$c_{j,m} \leqslant s_{j,m'}, \quad \forall j \in \mathcal{J}, \forall m, m' \in \mathcal{M} : (j,m) \prec (j,m'). \tag{4}$$

Variable linking

Variable linking: x and d

$$d_{j,m}^{p} \leqslant \min\{l^{p}, q_{j,m}\} \cdot x_{j,m}^{p}, \quad \forall (j,m) \in \mathcal{O}, \forall p \in \mathcal{P}. \tag{5}$$

l^p: length of period p

Variable linking

Variable linking: x and d

$$d^p_{j,m} \leqslant \min\{l^p, q_{j,m}\} \cdot x^p_{j,m}, \quad \forall (j,m) \in \mathcal{O}, \forall p \in \mathcal{P}. \tag{5}$$

l^p: length of period p

Variable linking: x, d and s

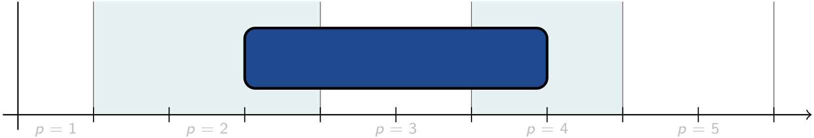
$$d^p_{j,m} \leqslant t^{p+1} - s_{j,m} + \gamma^p_{j,m} \cdot (1 - x^p_{j,m}), \quad \forall (j,m) \in \mathcal{O}, \forall p \in \mathcal{P}, \tag{6a}$$

$$d^p_{j,m} \leqslant c_{j,m} - t^p \cdot x^p_{j,m}, \quad \forall (j,m) \in \mathcal{O}, \forall p \in \mathcal{P}. \tag{6b}$$

These constraints guarantee **non-preemption**.

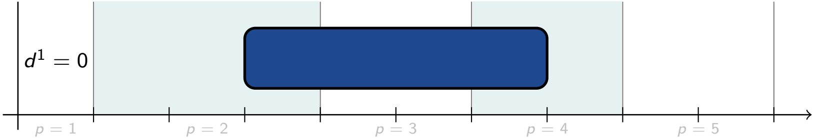
$[t^p, t^{p+1}]$: period p
$\gamma^p_{j,m}$: constant

Variable linking: numerical example



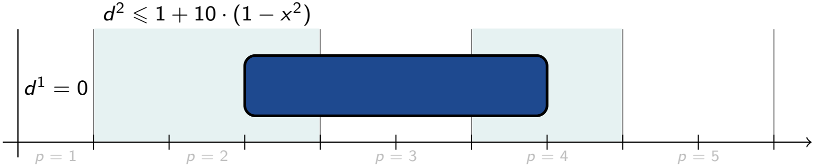
Variable linking: numerical example

- For p = 1, (6a) $\Rightarrow x^1 = 0 \Rightarrow d^1 = 0$,



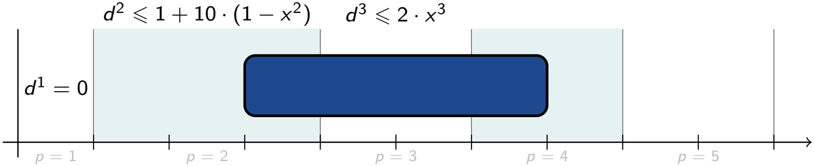
Variable linking: numerical example

- For p = 1, (6a) $\Rightarrow x^1 = 0 \Rightarrow d^1 = 0$,
- For p = 2, (6a) $\Rightarrow d^2 \leqslant 1 + 10 \cdot (1 - x^2)$,



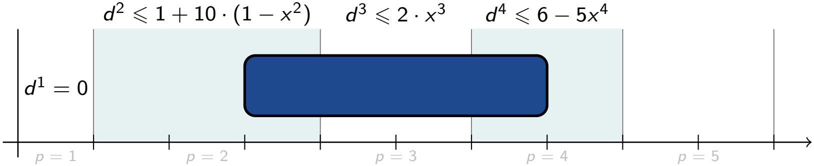
Variable linking: numerical example

- For p = 1, (6a) $\Rightarrow x^1 = 0 \Rightarrow d^1 = 0$,
- For p = 2, (6a) $\Rightarrow d^2 \leqslant 1 + 10 \cdot (1 - x^2)$,
- For p = 3, (5) $\Rightarrow d^3 \leqslant 2 \cdot x^3$



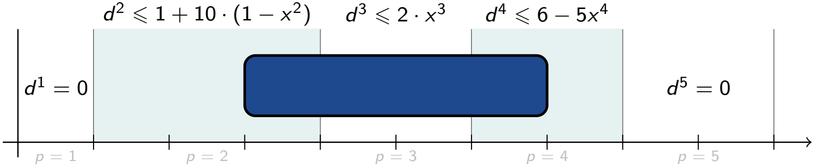
Variable linking: numerical example

- For p = 1, (6a) $\Rightarrow x^1 = 0 \Rightarrow d^1 = 0$,
- For p = 2, (6a) $\Rightarrow d^2 \leqslant 1 + 10 \cdot (1 - x^2)$,
- For p = 3, (5) $\Rightarrow d^3 \leqslant 2 \cdot x^3$
- For p = 4, (6b) $\Rightarrow d^4 \leqslant 6 - 5x^4$,



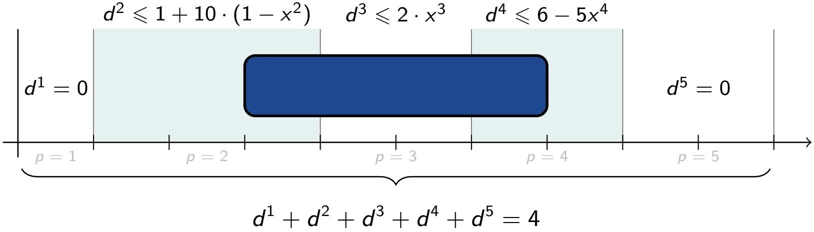
Variable linking: numerical example

- For p = 1, (6a) $\Rightarrow x^1 = 0 \Rightarrow d^1 = 0$,
- For p = 2, (6a) $\Rightarrow d^2 \leqslant 1 + 10 \cdot (1 - x^2)$,
- For p = 3, (5) $\Rightarrow d^3 \leqslant 2 \cdot x^3$
- For p = 4, (6b) $\Rightarrow d^4 \leqslant 6 - 5x^4$,
- For p = 5, (6b) $\Rightarrow x^5 = 0 \Rightarrow d^5 = 0$,



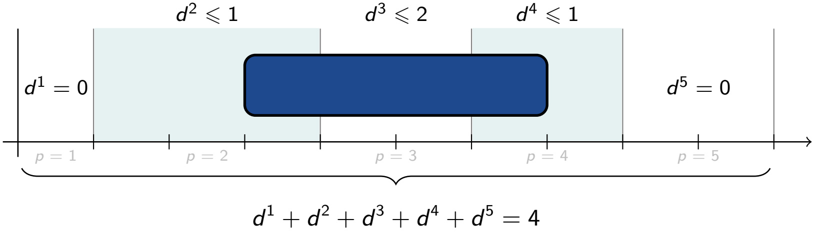
Variable linking: numerical example

- For p = 1, (6a) $\Rightarrow x^1 = 0 \Rightarrow d^1 = 0$,
- For p = 2, (6a) $\Rightarrow d^2 \leqslant 1 + 10 \cdot (1 - x^2)$,
- For p = 3, (5) $\Rightarrow d^3 \leqslant 2 \cdot x^3$
- For p = 4, (6b) $\Rightarrow d^4 \leqslant 6 - 5x^4$,
- For p = 5, (6b) $\Rightarrow x^5 = 0 \Rightarrow d^5 = 0$,
- (2) $\Rightarrow \sum_{p \in \mathcal{P}} d^p = q$



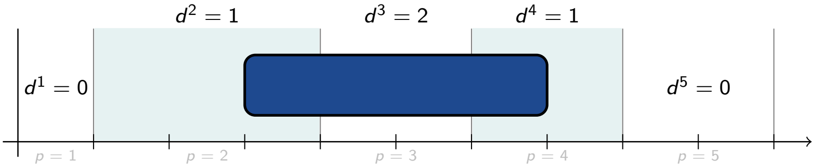
Variable linking: numerical example

- For p $= 1$, (6a) $\Rightarrow x^1 = 0 \Rightarrow d^1 = 0$,
- For p $= 2$, (6a) $\Rightarrow d^2 \leqslant 1 + 10 \cdot (1 - x^2)$,
- For p $= 3$, (5) $\Rightarrow d^3 \leqslant 2 \cdot x^3$
- For p $= 4$, (6b) $\Rightarrow d^4 \leqslant 6 - 5x^4$,
- For p $= 5$, (6b) $\Rightarrow x^5 = 0 \Rightarrow d^5 = 0$,
- (2) $\Rightarrow \sum_{p \in \mathcal{P}} d^p = q$



Variable linking: numerical example

- For p = 1, (6a) $\Rightarrow x^1 = 0 \Rightarrow d^1 = 0$,
- For p = 2, (6a) $\Rightarrow d^2 \leqslant 1 + 10 \cdot (1 - x^2)$,
- For p = 3, (5) $\Rightarrow d^3 \leqslant 2 \cdot x^3$
- For p = 4, (6b) $\Rightarrow d^4 \leqslant 6 - 5x^4$,
- For p = 5, (6b) $\Rightarrow x^5 = 0 \Rightarrow d^5 = 0$,
- (2) $\Rightarrow \sum_{p \in \mathcal{P}} d^p = q$



A period-indexed MILP formulation

$$
\begin{aligned}
\min \quad & \sum_{p \in \mathcal{P}} c^p \sum_{m \in M} \varphi_m \sum_{j \in \mathcal{J}} d^p_{j,m}, && && \text{(7a)}\\
[s.t.] \quad & \sum_{p \in \mathcal{P}} d^p_{j,m} = q_{j,m}, && \forall (j,m) \in \mathcal{O}, && \text{(7b)}\\
& d^p_{j,m} \leqslant \min\{l^p, q_{j,m}\} \cdot x^p_{j,m}, && \forall (j,m) \in \mathcal{O}, \forall p \in \mathcal{P}, && \text{(7c)}\\
& c_{j,m} - s_{j',m} \leqslant \alpha_{j,j',m} \cdot (1 - u_{j,j',m}), && \forall m \in \mathcal{M}, \forall j, j' \in \mathcal{J} : j < j', && \text{(7d)}\\
& c_{j',m} - s_{j,m} \leqslant \beta_{j,j',m} \cdot u_{j,j',m}, && \forall m \in \mathcal{M}, \forall j, j' \in \mathcal{J} : j < j', && \text{(7e)}\\
& d^p_{j,m} \leqslant t^{p+1} - s_{j,m} + \gamma^p_{j,m} \cdot (1 - x^p_{j,m}), && \forall (j,m) \in \mathcal{O}, \forall p \in \mathcal{P}, && \text{(7f)}\\
& d^p_{j,m} \leqslant c_{j,m} - t^p \cdot x^p_{j,m}, && \forall (j,m) \in \mathcal{O}, \forall p \in \mathcal{P}, && \text{(7g)}\\
& x^p_{j,m} \in \{0,1\}, d^p_{j,m} \geqslant 0, && \forall (j,m) \in \mathcal{O}, \forall p \in \mathcal{P}, && \text{(7h)}\\
& u_{j,j',m} \in \{0,1\}, s_{j,m} \geqslant 0, && \forall j, j' \in \mathcal{J} : j < j', \forall m \in \mathcal{M}. && \text{(7i)}
\end{aligned}
$$

Formulation properties

Natural date + disjunction variables / big-M formulation $\Rightarrow$ weak LP relaxations (see e.g. [14]).

Outline

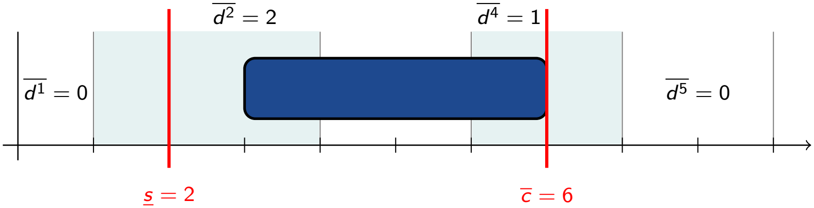
Tighter processing duration bounds

Let $\underline{s}_{j,m}$ denote the earliest starting and $\overline{c}_{j,m}$ the latest completion dates, and define

$$\overline{d}^p_{j,m} := \begin{cases} 0 & \text{if } t^{p+1} \leqslant \underline{s}_{j,m} \;\vee\; \overline{c}_{j,m} \leqslant t^p, \\ t^{p+1} - \underline{s}_{j,m} & \text{if } t^p \leqslant \underline{s}_{j,m} \leqslant t^{p+1}, \\ \overline{c}_{j,m} - t^p & \text{if } t^p \leqslant \overline{c}_{j,m} \leqslant t^{p+1}. \end{cases}$$

yielding

$$d^p_{j,m} \leqslant \min\{l_p, p_{j,m}, \overline{d}^p_{j,m}\} \cdot x^p_{j,m}, \quad \forall (j,m) \in \mathcal{O}, \forall p \in \mathcal{P}. \tag{8}$$

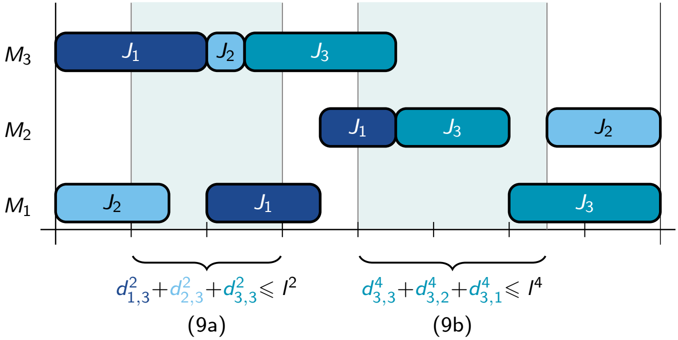


Non-overlap inequalities

- Same-machine (9a) and same-job (9b) operations non-overlap

$$\sum_{j \in \mathcal{J}} d_{j,m}^{p} \leqslant l^{p}, \qquad \forall m \in \mathcal{M}, \forall p \in \mathcal{P}, \tag{9a}$$

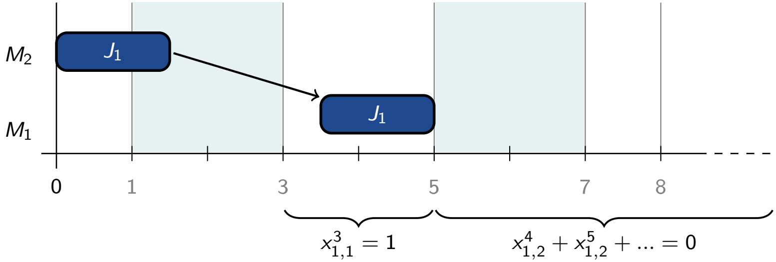
$$\sum_{m \in \mathcal{M}} d_{j,m}^{p} \leqslant l^{p}, \qquad \forall j \in \mathcal{J}, \forall p \in \mathcal{P}. \tag{9b}$$



Precedence inequalities

- No processing of predecessors* after current interval

$$\sum_{p'=p+1}^{|\mathcal{P}|} x_{j,m}^{p'} \leqslant (|\mathcal{P}| - p)(1 - x_{j,m'}^{p}), \quad \forall (j,m) \prec (j,m'), \forall p \leqslant |\mathcal{P}| - 1. \tag{10}$$



*A similar inequality holds for successors

Non-preemption inequalities

- No processing on the intervals after operation completion*

$$\sum_{p'=p+1}^{|\mathcal{P}|} x_{j,m}^{p'} \leqslant (|\mathcal{P}| - p)(1 - x_{j,m}^{p} + x_{j,m}^{p+1}), \quad \forall (j,m) \in \mathcal{O}, \forall p \leqslant |\mathcal{P}| - 1. \tag{11}$$

- No operation interrupts processing at intermediate intervals

$$x_{j,m}^{p} \geqslant x_{j,m}^{p-1} + x_{j,m}^{p+1} - 1, \quad \forall (j,m) \in \mathcal{O}, \forall p \in [\![2, |\mathcal{P}| - 1]\!]. \tag{12}$$

*A similar inequality holds for operation start

General inequalities

- Transitive precedence*

$$u_{j,j',m} + u_{j',j'',m} - 1 \leqslant u_{j,j'',m}, \quad \forall j, j', j'' \in \mathcal{J} : j < j' < j'', \forall m \in \mathcal{M}. \tag{13}$$

- Consecutive period processing

$$x^{p}_{j,m} + x^{p+1}_{j,m} + x^{p}_{j',m} + x^{p+1}_{j',m} \leqslant 3, \forall j, j' \in \mathcal{J} : j < j', \forall m \in \mathcal{M}, \forall p \leqslant |\mathcal{P}| - 1. \tag{14}$$

- Upper bound on number of processing intervals**

$$\sum_{p \in \mathcal{P}} x^{p}_{j,m} \leqslant \lfloor \frac{q_{j,m}}{\min_p l^p} \rfloor + 2, \quad \forall (j, m) \in \mathcal{O}. \tag{15}$$

* $u_{j,j',m} = 1$ if j precedes j' on m
**Similar to the trivial bin-packing relaxation

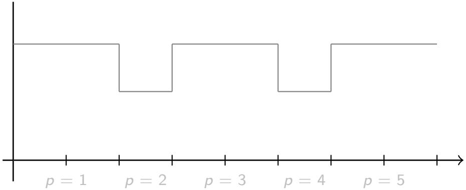
Outline

Instance features and settings

Subset of instances in Masmoudi et al. [5], based on:

- Classic benchmark JSSP instances
  - ft06 [15] with 6 machines and 6 jobs, $C_{max} = 56$,
  - la04 [16] with 5 machines and 10 jobs, $C_{max} = 590$.
- 3 time horizons $C = \lambda \cdot C_{max}$ with $\lambda \in \{1.0, 1.1, 1.2\}$,
- 5 sets of power values from $\mathcal{U}[5,10]$,
- On-off peak ToU profile [5]

$\Rightarrow$ 15 small and 15 large instances.



Gurobi on Intel Xeon Gold 6132 (1 thread, 3600s time-limit).

Valid inequalities

PI_0 : **no** valid inequalities

$\mathrm{PI}_{\mathrm{all}}$: **all** valid inequalities

PI_1 : **non-preemption**, **precedence** and **general** inequalities (10-15)

PI_2 : **tighter processing duration bounds** and **non-overlap** inequalities (8,9a,9b)

Valid inequalities

PI_0 : **no** valid inequalities

PI_{all} : **all** valid inequalities

PI_1 : **non-preemption**, **precedence** and **general** inequalities (10-15)

PI_2 : **tighter processing duration bounds** and **non-overlap** inequalities (8,9a,9b)

	PI_0			PI_1			PI_2			PI_{all}		
inst./λ	T/(%)	$\%^{root}_{bks}$	#nd	T/(%)	$\%^{root}_{bks}$	#nd	T/(%)	$\%^{root}_{bks}$	#nd	T/(%)	$\%^{root}_{bks}$	#nd
ft06/1.0	0.02	12.4%	1	0.02	12.4%	2	**0.01**	0.9%	1	0.01	0.9%	1
ft06/1.1	2.21	10.7%	2232	3.07	10.7%	2473	**0.40**	0.6%	470	0.62	0.6%	386
ft06/1.2	64.40	10.3%	50K	56.92	10.3%	38K	**1.96**	0.9%	1606	2.45	0.9%	1165
la04/1.0	20.62	12.6%	3910	23.14	12.6%	3700	**4.00**	0.5%	2237	6.44	0.5%	1900
la04/1.1	(2.8%)	11.8%	612K	(3.3%)	11.8%	276K	**0.18**	0.0%	75	0.53	0.0%	86
la04/1.2	(5.3%)	11.8%	506K	(5.4%)	11.8%	205K	**0.05**	0.0%	1	0.14	0.0%	1

Table: Comparison of the different proposed variants on the ft06 and la04 instances.

- Time to optimality/(gap% at time limit): **PI_2 is faster**

Valid inequalities

PI_0 : **no** valid inequalities

PI_{all} : **all** valid inequalities

PI_1 : **non-preemption**, **precedence** and **general** inequalities (10-15)

PI_2 : **tighter processing duration bounds** and **non-overlap** inequalities (8,9a,9b)

	PI_0			PI_1			PI_2			PI_{all}		
inst./λ	T/(%)	$\%_{bks}^{root}$	#nd	T/(%)	$\%_{bks}^{root}$	#nd	T/(%)	$\%_{bks}^{root}$	#nd	T/(%)	$\%_{bks}^{root}$	#nd
ft06/1.0	0.02	12.4%	1	0.02	12.4%	2	0.01	**0.9%**	1	0.01	0.9%	1
ft06/1.1	2.21	10.7%	2232	3.07	10.7%	2473	0.40	**0.6%**	470	0.62	0.6%	386
ft06/1.2	64.40	10.3%	50K	56.92	10.3%	38K	1.96	**0.9%**	1606	2.45	0.9%	1165
la04/1.0	20.62	12.6%	3910	23.14	12.6%	3700	4.00	**0.5%**	2237	6.44	0.5%	1900
la04/1.1	(2.8%)	11.8%	612K	(3.3%)	11.8%	276K	0.18	**0.0%**	75	0.53	0.0%	86
la04/1.2	(5.3%)	11.8%	506K	(5.4%)	11.8%	205K	0.05	**0.0%**	1	0.14	0.0%	1

Table: Comparison of the different proposed variants on the ft06 and la04 instances.

- Time to optimality/(gap% at time limit): PI_2 is faster
- Root relaxation strength: **PI_2 has strong root relaxations**

Valid inequalities

PI_0 : **no** valid inequalities

PI_{all} : **all** valid inequalities

PI_1 : **non-preemption**, **precedence** and **general** inequalities (10-15)

PI_2 : **tighter processing duration bounds** and **non-overlap** inequalities (8,9a,9b)

	PI_0			PI_1			PI_2			PI_{all}		
inst./λ	T/(%)	$\%_{bks}^{root}$	#nd	T/(%)	$\%_{bks}^{root}$	#nd	T/(%)	$\%_{bks}^{root}$	#nd	T/(%)	$\%_{bks}^{root}$	#nd
ft06/1.0	0.02	12.4%	1	0.02	12.4%	2	0.01	0.9%	1	0.01	0.9%	**1**
ft06/1.1	2.21	10.7%	2232	3.07	10.7%	2473	0.40	0.6%	470	0.62	0.6%	**386**
ft06/1.2	64.40	10.3%	50K	56.92	10.3%	38K	1.96	0.9%	1606	2.45	0.9%	**1165**
la04/1.0	20.62	12.6%	3910	23.14	12.6%	3700	4.00	0.5%	2237	6.44	0.5%	**1900**
la04/1.1	(2.8%)	11.8%	612K	(3.3%)	11.8%	276K	0.18	0.0%	75	0.53	0.0%	**86**
la04/1.2	(5.3%)	11.8%	506K	(5.4%)	11.8%	205K	0.05	0.0%	1	0.14	0.0%	**1**

Table: Comparison of the different proposed variants on the ft06 and la04 instances.

- Time to optimality/(gap% at time limit): PI_2 is faster
- Root relaxation strength: PI_2 has strong root relaxations
- Number of explored nodes: **PI_{all} requires slightly fewer nodes**

Time-Indexed vs Period-Indexed

PI_0 : no valid inequalities

$\mathrm{PI}_{\mathrm{all}}$: all valid inequalities

PI_1 : non-preemption, precedence and general inequalities

PI_2 : **tighter processing duration bounds** and **non-overlap** inequalities (8,9a,9b)

Time-Indexed vs Period-Indexed

PI_0 : no valid inequalities

PI_{all} : all valid inequalities

PI_1 : non-preemption, precedence and general inequalities

PI_2 : **tighter processing duration bounds** and **non-overlap** inequalities (8,9a,9b)

	TI			PI_2			relative $\left(\frac{PI_2}{TI}\right)$	
inst./λ	T/(%)	$\%^{root}_{bks}$	#nd	T/(%)	$\%^{root}_{bks}$	#nd	#cols	#rows
ft06/1.0	0.25	0.3%	1	**0.01**	0.9%	1	0.20	0.36
ft06/1.1	0.37	0.1%	1	**0.62**	0.6%	386	0.19	0.33
ft06/1.2	2.84	0.2%	16.4	**2.45**	0.9%	1165	0.20	0.35
la04/1.0	(0.63%)	0.5%	60	**6.44**	0.5%	1900	0.02	0.05
la04/1.1	1432.06	0.0%	8.6	**0.53**	0.0%	86	0.02	0.05
la04/1.2	2564.31	0.0%	27.6	**0.14**	0.0%	1	0.02	0.05

Table: Comparison of the PI_2 variant and the time-indexed model (TI) of Masmoudi et al. [5].

- Time to optimality/(gap% at time limit): **PI_2 is fastest**

Time-Indexed vs Period-Indexed

PI_0 : no valid inequalities

PI_1 : non-preemption, precedence and general inequalities

PI_{all} : all valid inequalities

PI_2 : **tighter processing duration bounds** and **non-overlap** inequalities (8,9a,9b)

	TI			PI_2			relative $\left(\frac{PI_2}{TI}\right)$	
inst./λ	T/(%)	$\%^{root}_{bks}$	#nd	T/(%)	$\%^{root}_{bks}$	#nd	#cols	#rows
ft06/1.0	0.25	**0.3%**	1	0.01	0.9%	1	0.20	0.36
ft06/1.1	0.37	**0.1%**	1	0.62	0.6%	386	0.19	0.33
ft06/1.2	2.84	**0.2%**	16.4	2.45	0.9%	1165	0.20	0.35
la04/1.0	(0.63%)	**0.5%**	60	6.44	0.5%	1900	0.02	0.05
la04/1.1	1432.06	**0.0%**	8.6	0.53	0.0%	86	0.02	0.05
la04/1.2	2564.31	**0.0%**	27.6	0.14	0.0%	1	0.02	0.05

Table: Comparison of the PI_2 variant and the time-indexed model (TI) of Masmoudi et al. [5].

- Time to optimality/(gap% at time limit): PI_2 is fastest
- Root relaxation strength: **comparable**

Time-Indexed vs Period-Indexed

PI_0 : no valid inequalities

PI_1 : non-preemption, precedence and general inequalities

PI_{all} : all valid inequalities

PI_2 : **tighter processing duration bounds** and **non-overlap** inequalities (8,9a,9b)

	TI			PI_2			relative $\left(\frac{PI_2}{TI}\right)$	
inst./λ	T/(%)	$\%^{root}_{bks}$	#nd	T/(%)	$\%^{root}_{bks}$	#nd	#cols	#rows
ft06/1.0	0.25	0.3%	1	0.01	0.9%	1	**0.20**	**0.36**
ft06/1.1	0.37	0.1%	1	0.62	0.6%	386	**0.19**	**0.33**
ft06/1.2	2.84	0.2%	16.4	2.45	0.9%	1165	**0.20**	**0.35**
la04/1.0	(0.63%)	0.5%	60	6.44	0.5%	1900	**0.02**	**0.05**
la04/1.1	1432.06	0.0%	8.6	0.53	0.0%	86	**0.02**	**0.05**
la04/1.2	2564.31	0.0%	27.6	0.14	0.0%	1	**0.02**	**0.05**

Table: Comparison of the PI_2 variant and the time-indexed model (TI) of Masmoudi et al. [5].

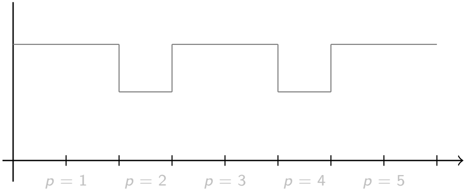
- Time to optimality/(gap% at time limit): PI_2 is fastest
- Root relaxation strength: comparable
- Model size: **PI_2 scales better**

Outline

Instance features and settings

Extended from instances in [5]:

- classic benchmark JSSP instances
  - **la04 with 5 machines and 10 jobs, $C_{\max}$= 590**.
- **Time horizon** $C = 1.2 \cdot C_{\max}$,
- 5 sets of power values from $\mathcal{U}[5,10]$,
- ToU profiles
  - **on-off peak (ratio 2:1),**
  - on-off peak (ratio 1:2),
  - on-off peak (ratio 1:1),
  - mid-off-on peak.

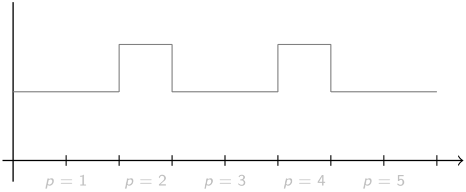


Gurobi on an Intel Xeon Gold 6132 (1 thread, 3600s time-limit).

Instance features and settings

Extended from instances in [5]:

- classic benchmark JSSP instances
  - » **la04 with 5 machines and 10 jobs, $C_{\max}$ = 590**.
- **Time horizon** $C = 1.2 \cdot C_{\max}$,
- 5 sets of power values from $\mathcal{U}[5,10]$,
- ToU profiles
  - » on-off peak (ratio 2:1),
  - » **on-off peak (ratio 1:2),**
  - » on-off peak (ratio 1:1),
  - » mid-off-on peak.

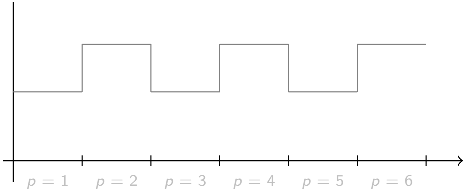


Gurobi on an Intel Xeon Gold 6132 (1 thread, 3600s time-limit).

Instance features and settings

Extended from instances in [5]:

- classic benchmark JSSP instances
  - **la04 with 5 machines and 10 jobs, $C_{\max}$= 590**.
- **Time horizon** $C = 1.2 \cdot C_{\max}$,
- 5 sets of power values from $\mathcal{U}[5,10]$,
- ToU profiles
  - on-off peak (ratio 2:1),
  - on-off peak (ratio 1:2),
  - **on-off peak (ratio 1:1),**
  - mid-off-on peak.

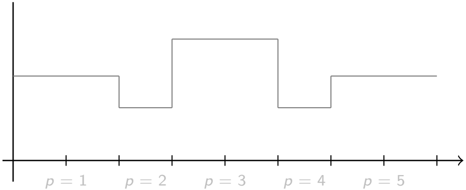


Gurobi on an Intel Xeon Gold 6132 (1 thread, 3600s time-limit).

Instance features and settings

Extended from instances in [5]:

- classic benchmark JSSP instances
  - **la04 with 5 machines and 10 jobs, $C_{\max}$= 590**.
- **Time horizon** $C = 1.2 \cdot C_{\max}$,
- 5 sets of power values from $\mathcal{U}[5,10]$,
- ToU profiles
  - on-off peak (ratio 2:1),
  - on-off peak (ratio 1:2),
  - on-off peak (ratio 1:1),
  - **mid-off-on peak.**



Gurobi on an Intel Xeon Gold 6132 (1 thread, 3600s time-limit).

ToU profile impact

ToU profile	TI T/(%)	TI $\%^{root}_{bks}$	TI #nd	PI_2 T/(%)	PI_2 $\%^{root}_{bks}$	PI_2 #nd
on-off (2:1)	2564.31	0.0%	28	**0.14**	0.0%	1
on-off (1:2)	3103.69	0.4%	925	**67.89**	1.0%	35K
on-off (1:1)	(0.11%)*	0.1%	407	**419.35**	0.1%	168K
mid-on-off	2406.81	0.5%	631	**84.92**	1.6%	12K

Table: Comparison of TI and PI_2 on different ToU profiles.

- Time to optimality/gap at time limit: **sensitivity to profile but PI_2 better**

*#opt. = 0

ToU profile impact

	TI			PI_2		
ToU profile	T/(%)	$\%_{\text{bks}}^{\text{root}}$	#nd	T/(%)	$\%_{\text{bks}}^{\text{root}}$	#nd
`on-off (2:1)`	2564.31	**0.0%**	28	0.14	0.0%	1
`on-off (1:2)`	3103.69	**0.4%**	925	67.89	1.0%	35K
`on-off (1:1)`	(0.11%)*	**0.1%**	407	419.35	0.1%	168K
`mid-on-off`	2406.81	**0.5%**	631	84.92	1.6%	12K

Table: Comparison of TI and PI_2 on different ToU profiles.

- Time to optimality/gap at time limit: sensitivity to profile but PI_2 better
- Root relaxation strength: **minor impact on both**

*#opt. = 0

ToU profile impact

	TI			PI_2		
ToU profile	T/(%)	$\%^{root}_{bks}$	#nd	T/(%)	$\%^{root}_{bks}$	#nd
`on-off (2:1)`	2564.31	0.0%	**28**	0.14	0.0%	**1**
`on-off (1:2)`	3103.69	0.4%	**925**	67.89	1.0%	**35K**
`on-off (1:1)`	(0.11%)*	0.1%	**407**	419.35	0.1%	**168K**
`mid-on-off`	2406.81	0.5%	**631**	84.92	1.6%	**12K**

Table: Comparison of TI and PI_2 on different ToU profiles.

- Time to optimality/gap at time limit: sensitivity to profile but PI_2 better
- Root relaxation strength: minor impact on both
- Number of explored nodes: **more time spent per node**

*#opt. = 0

ToU profile impact

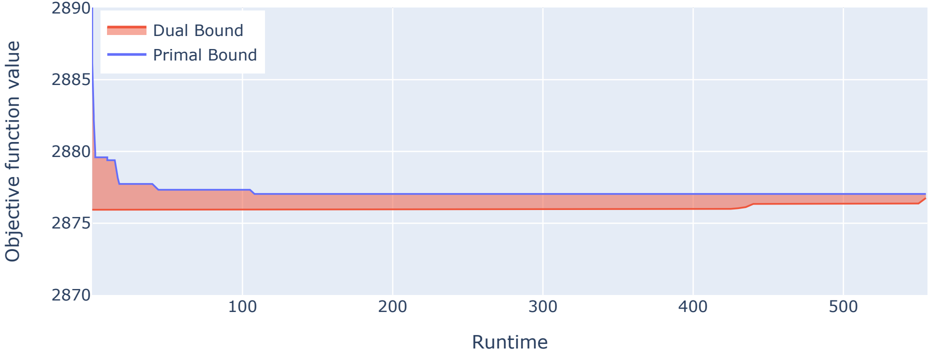


Figure: UB and LB progress on an example instance with on-off (1:1) profile

Outline

Conclusions and research perspectives

- A new model for the $Jm||\text{TEC}$, indexed on the ToU profile periods.
- Different families of valid inequalities were explored.
- Compared to the SoA Time-Indexed formulation, on benchmark instances:
  - ✓ more compact,
  - ✓ strong linear relaxations,
  - ✓ solves to optimality two open instances.
- Performance is sensitive to the ToU pricing profile.

Conclusions and research perspectives

- A new model for the $Jm||\text{TEC}$, indexed on the ToU profile periods.
- Different families of valid inequalities were explored.
- Compared to the SoA Time-Indexed formulation, on benchmark instances:
  - ✓ more compact,
  - ✓ strong linear relaxations,
  - ✓ solves to optimality two open instances.
- Performance is sensitive to the ToU pricing profile.

→ Studying period-indexed formulations relative to the ToU profile.

→ Characterizing strong valid inequalities for period-indexed formulations.

→ Generalizing to other shop-scheduling problems under ToU pricing.

References I

[1] Michael R Garey, David S Johnson, and Ravi Sethi. The complexity of flowshop and jobshop scheduling. *Mathematics of operations research*, 1(2):117–129, 1976.

[2] Clean Energy Alliance. Seasonal time-of-use pricing schemes, 2024. URL `https://web.archive.org/web/20241202083629/https://thecleanenergyalliance.org/time-of-use-pricing/`.

[3] R. Graham, E. Lawler, J. Lenstra, and A. Kan. Optimization and approximation in deterministic sequencing and scheduling: A survey. *Annals of Discrete Mathematics*, 5:287–326, 1977.

[4] Andreas Bley and Andreas Linß. Propagation and branching strategies for job shop scheduling minimizing the weighted energy consumption. In *International Conference on Operations Research*, pages 573–580. Springer, 2022.

[5] Oussama Masmoudi, Xavier Delorme, and Paolo Gianessi. Job-shop scheduling problem with energy consideration. *International Journal of Production Economics*, 216:12–22, 10 2019. ISSN 09255273. doi: 10.1016/j.ijpe.2019.03.021.

[6] Myoung Ju Park and Andy Ham. Energy-aware flexible job shop scheduling under time-of-use pricing. *International Journal of Production Economics*, 248, 6 2022. ISSN 09255273. doi: 10.1016/j.ijpe.2022.108507.

References II

[7] Enda Jiang and Ling Wang. Multi-objective optimization based on decomposition for flexible job shop scheduling under time-of-use electricity prices. *Knowledge-Based Systems*, 204, 9 2020. ISSN 09507051. doi: 10.1016/j.knosys.2020.106177.

[8] Minh Hung Ho, Faicel Hnaien, and Frederic Dugardin. Exact method to optimize the total electricity cost in two-machine permutation flow shop scheduling problem under time-of-use tariff. *Computers and Operations Research*, 144, 8 2022. ISSN 03050548. doi: 10.1016/j.cor.2022.105788.

[9] Mauro Gaggero, Massimo Paolucci, and Roberto Ronco. Exact and heuristic solution approaches for energy-efficient identical parallel machine scheduling with time-of-use costs. *European Journal of Operational Research*, 311:845–866, 12 2023. ISSN 03772217. doi: 10.1016/j.ejor.2023.05.040.

[10] Ada Che, Shibohua Zhang, and Xueqi Wu. Energy-conscious unrelated parallel machine scheduling under time-of-use electricity tariffs. *Journal of Cleaner Production*, 156:688–697, 7 2017. ISSN 09596526. doi: 10.1016/j.jclepro.2017.04.018.

[11] Jian Ya Ding, Shiji Song, Rui Zhang, Raymond Chiong, and Cheng Wu. Parallel machine scheduling under time-of-use electricity prices: New models and optimization approaches. *IEEE Transactions on Automation Science and Engineering*, 13:1138–1154, 4 2016. ISSN 15455955. doi: 10.1109/TASE.2015.2495328.

References III

[12] Zheng Tian and Li Zheng. Single machine parallel-batch scheduling under time-of-use electricity prices: New formulations and optimisation approaches. *European Journal of Operational Research*, 312:512–524, 1 2024. ISSN 03772217. doi: 10.1016/j.ejor.2023.07.012.

[13] Junheng Cheng, Feng Chu, Chengbin Chu, and Weili Xia. Bi-objective optimization of single-machine batch scheduling under time-of-use electricity prices. *RAIRO - Operations Research*, 50:715–732, 10 2016. ISSN 28047303. doi: 10.1051/ro/2015063.

[14] David Applegate and William Cook. A computational study of the job-shop scheduling problem. *ORSA Journal on computing*, 3(2):149–156, 1991. doi: 10.1287/ijoc.3.2.149.

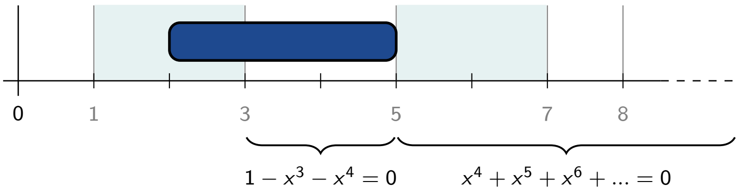
[15] H Fisher and GL Thompson. Probabilistic learning combinations of local job-shop scheduling rules. *Prentice Hall, Englewood Cliffs, New Jersey*, pages 225–251, 1963.

[16] Stephen Lawrence. Resouce constrained project scheduling: An experimental investigation of heuristic scheduling techniques (supplement). *Graduate School of Industrial Administration, Carnegie-Mellon University*, 1984.

Non-preemption inequalities (1)

- No processing on the intervals after operation completion*

$$\sum_{p'=p+1}^{|\mathcal{P}|} x_{j,m}^{p'} \leqslant (|\mathcal{P}| - p)(1 - x_{j,m}^{p} + x_{j,m}^{p+1}), \quad \forall (j,m) \in \mathcal{O}, \forall p \leqslant |\mathcal{P}| - 1. \tag{16}$$



*A similar inequality holds for operation start

Non-preemption inequalities (2)

- No operation interrupts processing

$$x^{p}_{j,m} \geqslant x^{p-1}_{j,m} + x^{p+1}_{j,m} - 1, \quad \forall (j,m) \in \mathcal{O}, \forall p \in [\![2, |\mathcal{P}| - 1]\!]. \tag{17}$$